Single- and Multi-user Adaptive Pragmatic Trellis Coded Modulation for OFDM Systems

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Abstract--**OFDM has recently been considered as one of the main next-generation broadband wireless access solutions because of its robustness over heavily impaired links and its high spectral efficiency. In this paper, OFDM-based adaptive TCM schemes in both single- and multi-user environments are proposed. The adaptive scheme employs the family of pragmatic TCM to combine different coded modulation modes into one codeword while reducing the hardware complexity. A set of switching levels is analytically derived to determine the appropriate modes for each value of the received SNR. The error and throughput performance of the proposed system is analyzed and it is shown that adaptive single user TCM results in considerable performance improvement. It is also demonstrated that by jointly optimizing the overall spectral efficiency while guaranteeing a minimum data rate and a desired BER level to each user, the spectral or power efficiency can be further improved in the multi-user environment.**

I. INTRODUCTION

Over the last decade, there has been an explosion of cellular and wireless communication systems whose ultimate goal is to provide universal and ubiquitous personal services without regards to mobility or location. To support broadband wireless communications, advanced technologies for increasing system capacities and for mitigating the destructive effects of the wireless channel are needed. Orthogonal Frequency Division Multiplexing (OFDM) is a promising solution for supporting broadband wireless and multimedia communications in the next-generation wireless systems. It has long been regarded as an efficient approach to combat the adverse effects of the wireless channel [1, 2]. Its inherent multicarrier nature also allows the use of adaptive transmission to significantly enhance the system performance and more importantly to realize robust and reliable high rate multimedia and evolving Internet services. Significant work has revealed that when the transmitter has knowledge of the channel state information (CSI), considerable performance gain can be achieved by adjusting transmit power, constellation size, code rate, or any combinations of these parameters [1, 3-9]. In a multiuser environment, further improvement can be achieved by dynamically allocating resources, such as subcarriers and power among the users [10, 11].

Trellis coded modulation (TCM) is a combined coding and modulation scheme for improving the reliability of a communication system without increasing the transmitted power or required bandwidth [12, 13]. In this paper, we propose adaptive TCM schemes for OFDM in both single-user and multiuser environments. Single-user, single-carrier adaptive TCM has been studied in [6, 7, 9]. In [6], the authors showed that the Ungerboeck code that is designed for the additive white Gaussian noise (AWGN) channel can be superimposed on adaptive modulation for fading channels, with the same approximate coding gains. In [7, 9], the family of pragmatic TCM [14] is applied for adaptation in flat fading channels. Unlike [6, 7, 9], frequency selective fading channels are considered in our paper. In this case, OFDM is applied to deal with the effects of frequency selectivity. OFDM-based adaptive TCM has been previously studied in [8]. However, optimal Ungerboeck TCM used in [6,8] uses different generator for different modulation levels. As a result, the modulation level cannot be changed within one codeword. This actually limits the codeword length to the channel coherence time [6] or coherence bandwidth [8], which leads to an inferior performance when the codeword length is too short. In contrast, pragmatic TCM is applied in our algorithm because it is particularly well suited for adaptive coded modulation. Different modulation levels can be included in one codeword by means of an adaptive trellis. Thus, the code length is no longer limited by the channel coherence time or bandwidth. Moreover, the same Viterbi decoder can be used at the receiver to reduce the hardware complexity.

In single user adaptive TCM, a very low constellation size will be assigned to subcarriers in deep fades to avoid bit errors, and this results in inefficient spectral utilization. In a multiuser environment, however, a subcarrier that appears to be in deep fade to one user is unlikely to be in deep fades to some other users [10, 11]. This motivates us to dynamically allocate the subcarriers to users, instead of allocating users fixed frequency bands such as in FDMA or time slots such as in TDMA. Hence and in contrast to [6-9], this paper also extends adaptive TCM to a multiuser environment. In the proposed scheme, the subcarriers are dynamically allocated to the users according to their instantaneous channel conditions, with the objective of maximizing the overall spectral efficiency while guaranteeing a minimum data rate and a

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desired BER (bit error rate) level to each user. A practical algorithm based on the special structure of the allocation problem is proposed to solve the optimization problem. In particular, the analytical expressions for the system performance in terms of SER (sequence error rate) and BER along with throughput in terms of BPS (bits per symbol) are derived in the single user environment. Based on the BER and BPS expressions, a set of optimal switching levels is obtained by optimizing a cost function. Compared to the conservative switching levels adopted by the previous papers, e.g. [7, 8, 9], the switching levels obtained in this paper are able to offer higher throughput, while keeping the actual BER around the pre-set target BER.

The rest of this paper is organized as follows. In Section II, the family of pragmatic TCM is introduced, and its applicability to adaptive systems is explained. In Section III, the single user adaptive TCM scheme is proposed. The performance is analyzed theoretically, and optimal switching levels are derived. Section IV presents the adaptive TCM scheme in a multiuser environment. The performance of the proposed systems is demonstrated in Section V. Finally, conclusions are given in Section VI.

II. PRAGMATIC TCM AND ADAPTIVE TRELLIS

The family of pragmatic TCM has been proposed by Viterbi *et al.* in [14]. Such TCM scheme can have an almost identical performance as the best Ungerboeck code [12] while being simpler and more flexible. As shown in Fig. 1, the pragmatic approach uses a core rate ½ optimal binary convolutional encoder. If there are *k* input bits, then 1 bit is coded into 2 and *k*-1 bits remains uncoded. The output bits are mapped into an MQAM (M-ary Quadrate Amplitude Modulation) symbol according to a slightly modified mapping scheme from Ungerboeck's set partition mapping [12]. It was shown in [7] that the optimal generator for binary convolutional code and double-gray mapping provide optimal performance for pragmatic TCM. In double-gray mapping, the coded bits choose a coset according to the Gray code, and the uncoded bits in every coset are Gray mapped. Double-gray mapping will be used throughout this paper, unless otherwise stated.

As mentioned in Section I, the family of pragmatic codes is particularly well suited for adaptive coding. The core convolutional code remains fixed and the number of uncoded bits varies according to the channel. Therefore, we have a trellis with a fixed number of states but a varying number of parallel branches between each transition step, as illustrated in Fig. 2. The obvious advantage of the adaptive trellis structure is that various coded modulation modes can be included in one codeword, and the codeword length is no longer limited by the channel coherence time or bandwidth. In contrast, if the Ungerboeck TCM is used, the coded modulation mode can only be changed from frame to frame, rather than from symbol to symbol, and the codeword length is limited by the channel varying speed. When decoding, the points within each coset that have most likely been transmitted are first determined. From these points the maximum-likelihood coset sequence is then calculated.

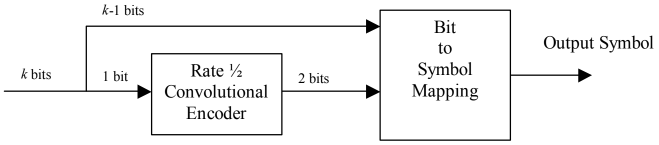

Fig. 1: Pragmatic TCM

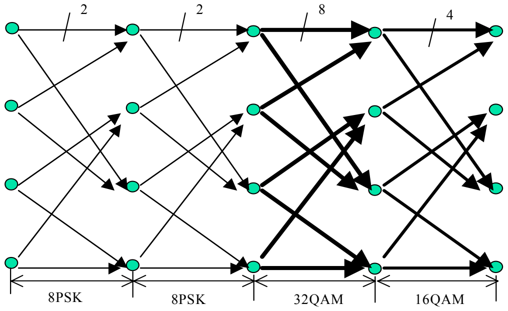

Fig. 2: Adaptive trellis of pragmatic TCM

III. SINGLE USER ADAPTIVE TCM

3.1 Block Diagram

The simplified block diagram of the single user adaptive TCM is shown in Fig. 3. Consider an OFDM system with *N* subcarriers. Assume that CSI is available at the receiver after channel estimation, and is sent back to the transmitter via a feedback channel. In this paper, the effects of estimation errors and feedback delay are ignored, i.e., perfect CSI is assumed to be available at the transmitter. The effect of inaccurate channel estimation and noisy feedback has been studied in [9], and the adaptive TCM system was shown to be robust to inaccuracy of CSI. At the transmitter, *k* information bits are fed into a TCM encoder to form an MQAM symbol. 1 information bits are encoded into 2 by a rate ½ convolutional encoder. These 2 bits are used to choose one of the 2^2 cosets from a partition of the signal constellation. The remaining *k*-1 bits are uncoded and are used to choose one of the signal points in the selected coset. The number *k* on each subcarrier is adapted to the instantaneous CSI on it. The adaptation decision is sent to the receiver via a control channel and the receiver is able to decode the received codeword according to this information. In this paper, the power allocated to every bit is fixed. This is actually a practical counterpart of the waterfilling technique, because subcarriers with high channel gain are allocated more bits and hence more power, and vice versa.

As was mentioned, we should decide which constellation size to transmit, i.e. the number of information bits *k*, for each value of received *SNR*. Assume that there are *J* constellation sizes M_0, M_1, …, M_{J-1}, where M_0 corresponds to no transmission and M_{J-1} corresponds to the largest constellation size. Let $\mu_0, \mu_1, \mu_2, \ldots, \mu_{J-1}, \mu_J$ denote the switching levels, where $\mu_0 = 0$ and $\mu_J = \infty$. Constellation size M_j is selected when the received *SNR* falls in the interval (μ_j, μ_{j+1}). The switching levels should be selected carefully, so that the resulting BER is around a pre-set target BER, and the throughput is as high as possible. In Subsection 3.3, a set of appropriate switching levels is obtained by solving an optimization problem.

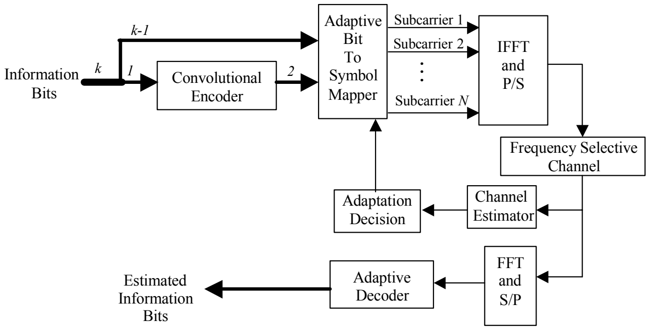

Fig. 3: System diagram of the OFDM-based single user adaptive TCM

3.2 Performance Analysis

The performance of the proposed scheme is analyzed theoretically in this section. Assume that the same ideal interleaver as in [7] is applied, so that the encoded symbols are faded by uncorrelated fadings. Let L be the length of an error event. $\mathbf{C}_L = (\mathbf{c}_1, \mathbf{c}_2, ..., \mathbf{c}_L)$ is the sequence of binary vectors that are fed into a mapper and the mapper outputs a sequence of encoded symbols $\mathbf{X}_L = (x_1, x_2, ..., x_L)$. $\mathbf{E}_L = (\mathbf{e}_1, \mathbf{e}_2, ..., \mathbf{e}_L)$ is a sequence of binary error vectors. Let $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, ..., \alpha_L)$ be the corresponding fading amplitudes. k_i is the number of information bits of the i^{th} symbol in a code sequence.

The bound on SER is calculated as

$$\begin{aligned} P(e) &\le \sum_{L=1}^{\infty} \sum_{\mathbf{X}_L} \sum_{\mathbf{X'}_L \neq \mathbf{X}_L} P(\mathbf{X}_L) P(\mathbf{X}_L \rightarrow \mathbf{X'}_L) \\ &= \sum_{L=1}^{\infty} \sum_{\mathbf{C}_L} \sum_{\mathbf{E}_L \neq 0} P(\mathbf{C}_L) E_{\boldsymbol{\alpha}}(P(\mathbf{C}_L \rightarrow \mathbf{C}_L \oplus \mathbf{E}_L \mid \boldsymbol{\alpha})) \\ &= T(\frac{E_b}{N_o}, I)\big|_{I=1} \end{aligned} \tag{1}$$

The right hand side of the above equation is the transfer function of the error state diagram where I corresponds to the incorrect input bits associated with each error vector [13]. As an example, the error state diagram of the pragmatic TCM of constraint length 3 has the structure shown in Fig. 4. The corresponding transfer function can be calculated as

$$T(\frac{E_b}{N_o}, I) = \frac{1}{1 - g_4 g_5}[\frac{g_1 g_2 g_3 g_7}{(1 - g_4 g_5)(1 - g_6) - g_2 g_3 g_5} + g_1 g_4 g_7] \tag{2}$$

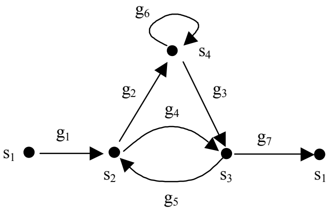

Fig. 4: Error state diagram

In order to calculate the transfer function, the labeling on each branch for the proposed adaptive pragmatic TCM should be derived first. The pair error probability is

$$\begin{aligned} P(\mathbf{C}_L \rightarrow \mathbf{C}_L \oplus \mathbf{E}_L) &= E_{\boldsymbol{\alpha}}(P(\mathbf{C}_L \rightarrow \mathbf{C}_L \oplus \mathbf{E}_L \mid \boldsymbol{\alpha})) \\ &\le E_{\boldsymbol{\alpha}}(\exp(-\frac{1}{4N_o}\sum_{i=1}^{L} k_i E_b \alpha_i^2 \parallel f_{M_i}(\mathbf{c}_i) - f_{M_i}(\mathbf{c}_i \oplus \mathbf{e}_i) \parallel^2)) \\ &= E_{\boldsymbol{\alpha}}(\prod_{i=1}^{L} \exp(-\frac{k_i E_b}{4N_o} \alpha_i^2 \parallel f_{M_i}(\mathbf{c}_i) - f_{M_i}(\mathbf{c}_i \oplus \mathbf{e}_i) \parallel^2)) \end{aligned} \tag{3}$$

Substituting (3) into (1) yields

$$P(e) \le \sum_{L=1}^{\infty} \sum_{\mathbf{E}_L \neq 0} E_{\boldsymbol{\alpha}}(W(\mathbf{E}_L \mid \boldsymbol{\alpha})) \tag{4}$$

where

$$W(\mathbf{E}_L \mid \boldsymbol{\alpha}) = \sum_{\mathbf{C}_L} P(\mathbf{C}_L) \prod_{i=1}^{L} \exp(-\frac{k_i E_b}{4N_o} \alpha_i^2 \parallel f_{M_i}(\mathbf{c}_i) - f_{M_i}(\mathbf{c}_i \oplus \mathbf{e}_i) \parallel^2) \tag{5}$$

Since the family of pragmatic TCM is isometric, and the labels of the branches of the error state diagrams are scalars [13], (4) can be written as

$$\begin{aligned} P(e) &\le \sum_{L=1}^{\infty} \sum_{\mathbf{E}_L \neq 0} \prod_{i=1}^{L} E_{\boldsymbol{\alpha}}(W(\mathbf{e}_i \mid \boldsymbol{\alpha})) \\ &= \sum_{L=1}^{\infty} \sum_{\mathbf{E}_L \neq 0} \prod_{i=1}^{L} E_{\boldsymbol{\alpha}}(2^{-k_i} \sum_{\mathbf{c}_i = \mathbf{c}(v)} \exp(-\frac{k_i E_b}{4N_o} \alpha_i^2 \parallel f_{M_i}(\mathbf{c}_i) - f_{M_i}(\mathbf{c}_i \oplus \mathbf{e}_i) \parallel^2)) \\ &= \sum_{L=1}^{\infty} \sum_{\mathbf{E}_L \neq 0} \prod_{i=1}^{L} W(\mathbf{e}_i) \end{aligned} \tag{6}$$

where $\mathbf{c}(v)$ denotes the vector $\mathbf{c}$ with its v^{th} component chosen as equal to either 1 or 0, arbitrarily. $W(\mathbf{e})$ is called the error weight profile. The labeling on each branch is then derived

$$\begin{aligned} g &= E_{\boldsymbol{\alpha}}(\sum_{q_{\boldsymbol{\alpha}}} (W(\mathbf{e}_{i,q_\alpha} \mid \boldsymbol{\alpha}) I^{\varepsilon_{q\alpha}}) \\ &= E_{\boldsymbol{\gamma}}(\sum_{q_{\boldsymbol{\gamma}}} (W(\mathbf{e}_{i,q_{\boldsymbol{\gamma}}} \mid \boldsymbol{\gamma}) I^{\varepsilon_{q_{\boldsymbol{\gamma}}}}) \end{aligned} \tag{7}$$

where the summation is due to parallel transition, and $\gamma = \alpha^2 \frac{E_b}{N_o}$ is the received *SNR*.

If the channel is assumed to be Rayleigh fading, then

$$f(\alpha) = 2\alpha \exp(-\alpha^2) \tag{8}$$

$$f(\gamma) = \frac{1}{E_b / N_o} \exp(-\frac{\gamma}{E_b / N_o}) \tag{9}$$

$$\begin{aligned} g = &\int_{\mu_1}^{\mu_2} \frac{1}{2} \sum_{\mathbf{c}=\mathbf{c}_4(v)} \exp(-\frac{\gamma}{4} \parallel f_4(\mathbf{c}) - f_4(\mathbf{c} \oplus \mathbf{e}) \parallel^2) I^{\varepsilon} \frac{1}{E_b / N_o} \exp(-\frac{\gamma}{E_b / N_o}) d\gamma \\ &+ \int_{\mu_2}^{\mu_3} \sum_{q=1}^{2} \frac{1}{4} \sum_{\mathbf{c}=\mathbf{c}_8(v)} \exp(-\frac{2\gamma}{4} \parallel f_8(\mathbf{c}) - f_8(\mathbf{c} \oplus \mathbf{e}_q) \parallel^2) I^{\varepsilon_q} \frac{1}{E_b / N_o} \exp(-\frac{\gamma}{E_b / N_o}) d\gamma \\ &+ \int_{\mu_3}^{\mu_4} \sum_{q=1}^{4} \frac{1}{8} \sum_{\mathbf{c}=\mathbf{c}_{16}(v)} \exp(-\frac{3\gamma}{4} \parallel f_{16}(\mathbf{c}) - f_{16}(\mathbf{c} \oplus \mathbf{e}_q) \parallel^2) I^{\varepsilon_q} \frac{1}{E_b / N_o} \exp(-\frac{\gamma}{E_b / N_o}) d\gamma \\ &+ \int_{\mu_4}^{\mu_5} \sum_{q=1}^{8} \frac{1}{16} \sum_{\mathbf{c}=\mathbf{c}_{32}(v)} \exp(-\frac{4\gamma}{4} \parallel f_{32}(\mathbf{c}) - f_{32}(\mathbf{c} \oplus \mathbf{e}_q) \parallel^2) I^{\varepsilon_q} \frac{1}{E_b / N_o} \exp(-\frac{\gamma}{E_b / N_o}) d\gamma \\ &+ \int_{\mu_5}^{\infty} \sum_{q=1}^{16} \frac{1}{32} \sum_{\mathbf{c}=\mathbf{c}_{64}(v)} \exp(-\frac{5\gamma}{4} \parallel f_{64}(\mathbf{c}) - f_{64}(\mathbf{c} \oplus \mathbf{e}_q) \parallel^2) I^{\varepsilon_q} \frac{1}{E_b / N_o} \exp(-\frac{\gamma}{E_b / N_o}) d\gamma \end{aligned} \tag{10}$$

After appropriate labeling, the transfer function can be calculated. The BER of the TCM system is then given by

$$P_b \le P_{\parallel} + \frac{1}{n} \frac{\partial T(\frac{E_b}{N_o}, I)}{\partial I}\Big|_{I=1} \tag{11}$$

where $P_{\|}$ is the probability of errors due to the parallel transitions, and in our algorithm,

$$\begin{aligned}
P_{\|} &= \int_{\mu_2}^{\mu_3} \frac{1}{4} erfc(\sqrt{\frac{2E_b}{N_o}}) \frac{1}{E_b/N_o} \exp(-\frac{\gamma}{E_b/N_o}) d\gamma \\
&+ \int_{\mu_3}^{\mu_4} \frac{1}{3} erfc(\sqrt{\frac{1.2E_b}{N_o}}) \frac{1}{E_b/N_o} \exp(-\frac{\gamma}{E_b/N_o}) d\gamma \\
&+ \int_{\mu_4}^{\mu_5} \frac{1}{32} [10 erfc(\sqrt{\frac{0.8E_b}{N_o}}) + 10 erfc(\sqrt{\frac{1.6E_b}{N_o}}) + 3.5 erfc(\sqrt{\frac{3.2E_b}{N_o}}) \\
&\quad + 6 erfc(\sqrt{\frac{4E_b}{N_o}}) + erfc(\sqrt{\frac{6.4E_b}{N_o}})] \frac{1}{E_b/N_o} \exp(-\frac{\gamma}{E_b/N_o}) d\gamma \\
&+ \int_{\mu_5}^{\infty} \frac{3}{10} erfc(\sqrt{\frac{10E_b}{21N_o}}) \frac{1}{E_b/N_o} \exp(-\frac{\gamma}{E_b/N_o}) d\gamma
\end{aligned} \tag{12}$$

with $\bar{n}$ denoting the average throughput. That is,

$$\begin{aligned}
\bar{n} &= \int_{\mu_1}^{\mu_2} \frac{1}{E_b/N_o} \exp(-\frac{\gamma}{E_b/N_o}) d\gamma + 2\int_{\mu_2}^{\mu_3} \frac{1}{E_b/N_o} \exp(-\frac{\gamma}{E_b/N_o}) d\gamma \\
&+ 3\int_{\mu_3}^{\mu_4} \frac{1}{E_b/N_o} \exp(-\frac{\gamma}{E_b/N_o}) d\gamma + 4\int_{\mu_4}^{\mu_5} \frac{1}{E_b/N_o} \exp(-\frac{\gamma}{E_b/N_o}) d\gamma \\
&+ 5\int_{\mu_5}^{\infty} \frac{1}{E_b/N_o} \exp(-\frac{\gamma}{E_b/N_o}) d\gamma
\end{aligned} \tag{13}$$

3.3 Switching Levels

In [15], Torrance and Hanzo derived the optimal switching levels for adaptive modulation. In this subsection, we will derive switching levels for adaptive pragmatic TCM following a similar idea, but with some modifications. As was mentioned in Subsection 3.1, the switching levels are to decide which constellation size to use based on the received *SNR* value. The expressions on BER and throughput were derived in the previous subsection and both of them depend on the switching levels. The difference between the actual measured performance and the desired performance is defined as cost functions. The optimal switching levels are obtained by minimization of the cost functions. For a given average E_b/N_o value, the cost function is defined as

$$\varphi_{overall} = \varphi_{BER} + \rho \varphi_{BPS} \tag{14}$$

where ρ is a parameter to adjust the weight for different values of E_b/N_o. Thus, we have different switching levels for different E_b/N_o. In contrast, the idea in [15] is to derive fixed switching levels for the whole *SNR* range. Thus, the resultant BER drops at high E_b/N_o values, implying that the throughput can be potentially further improved. The introduction of ρ is to allow a BPS improvement at high E_b/N_o. Let

$$\varphi_{BER} = \begin{cases} 10\log(\frac{BER_m}{BER_d}) & BER_m > BER_d \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

$$\varphi_{BPS} = \begin{cases} BPS_d - BPS_m & BPS_d < BPS_m \\ 0 & \text{othwise} \end{cases} \tag{16}$$

where BER_m, BER_d, BPS_m, and BPS_d is the measured and desired BER and BPS respectively. Minimization of the cost function can be achieved by an iterative nonlinear search.

Since the BER expression derived for adaptive TCM is extremely complex, we approximate BER_m by a simplified formula (17), under the assumption that the modulation mode changes codeword by codeword, rather than symbol by symbol. Specifically, we use

$$\begin{aligned}
BER_m &= \bar{n}^{-1} [\int_{\mu_1}^{\mu_2} BER_{QPSK} f(\gamma) d\gamma + 2\int_{\mu_2}^{\mu_3} BER_{8PSK} f(\gamma) d\gamma \\
&+ 3\int_{\mu_1}^{\mu_2} BER_{16QAM} f(\gamma) d\gamma + 4\int_{\mu_4}^{\mu_5} BER_{32QAM} f(\gamma) d\gamma \\
&+ 5\int_{\mu_5}^{\infty} BER_{64QAM} f(\gamma) d\gamma]
\end{aligned} \tag{17}$$

The BER performance for every mode is obtained by simulation in AWGN channel. Since the simplified BER formula is used for optimization, it cannot be ensured that the resultant BER is exactly at the desired level. Usually at low *SNR*, the actual BER is a little higher. Furthermore, at high *SNR*, the actual BER is lower.

IV. MULTIUSER ADAPTIVE TCM

In this section, we present a multiuser adaptive TCM scheme that can effectively exploit multiuser diversity and enhance the system throughput. This scheme works in the downlink, where the base station (BS) allocates appropriate groups of subcarriers to the users. Then, the coding on the allocated subcarriers of every user is almost the same as the single user adaptive TCM. Assume that there are *U* users in total. *N* subcarriers are dynamically allocated to the users and appropriate coded modulation modes are decided at each subcarrier according to the instantaneous CSI. The purpose of multiuser adaptive TCM is to maximize the overall throughput, while satisfying each user's minimum rate requirement. Again, the power allocated to every transmitted bit is fixed.

The objective is formulated into a linear programming problem:

$$\max_{c_{u,n}, \rho_{u,n}} \sum_{u=1}^{U} \sum_{n=1}^{N} \rho_{u,n} c_{u,n} \tag{18}$$

subject to: $\sum_{n=1}^{N} \rho_{u,n} c_{u,n} \geq r_u \quad \forall u$

and if $\rho_{u',n} = 1$, then $\rho_{u,n} = 0 \; \forall u \neq u'$

$\rho_{u,n}$ is the assignment indicator variable for the u^{th} user and the n^{th} subcarrier, and is either 1 or 0, denoting the subcarrier *n* is allocated to user *u* or not. r_u is the minimum rate requirement of user *u*. $c_{u,n}$ is the number of information bits the n^{th} subcarrier can carry if it is allocated to user *u*. $c_{u,n}$ depends on the desired BER, the transmitted power and the channel gain user *u* on subcarrier *n*.

We will propose in the following a real time allocation scheme based on the special structure of this particular problem. At first, each subcarrier is allocated to the user with the highest channel gain on it. Since a subcarrier with a higher channel gain will be loaded with more bits, this allocation will maximize the overall throughput regardless of the users' minimum rate requirements. The subcarriers will then need to be reallocated to the users whose rate requirements are not yet satisfied. To reach the constrained optimal solution, every reallocation should cause the least decrease in the overall throughput, while the number of reallocation operations should be minimized. To realize it, we define a cost function for each reallocation. The cost function is proportional to the decrease in the overall throughput, and inversely proportional to the increase of the data rate of the user the subcarrier is reallocated to. That is,

$$e_{u_i,n} = \frac{(c_{u^*,n} - c_{u,n})}{c_{u,n}} \qquad \forall n \tag{19}$$

is the cost function of reallocating a subcarrier that is originally assigned to user u^* to user u.

The multiuser dynamic allocation is mathematically described as follows:

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Step 1:
For subcarrier $n = 1:N$
    $u^* = \arg\max_{u}(g_{u,n})$
    $\rho_{u^*,n} = 1,\ \rho_{u,n} = 0\ \forall u \neq u^*$
End
Step2:
For all u's satisfying $\sum_{n=1}^{N} \rho_{u,n} c_{u,n} < r_u$
    calculate cost functions $e_{u,n} = \frac{(c_{u^*,n} - c_{u,n})}{c_{u,n}}\ \forall n$
    While $\sum_{n=1}^{N} \rho_{u,n} c_{u,n} < r_u$
        $n' = \arg\min_{n} e_{u,n}$
        If $\sum_{n=1}^{N} c_{u^*,n} - c_{u^*,n'} \geq r_{u^*}$
            Reallocate subcarrier $n'$ to user u,
        End
    End
End
```

V. SIMULATION RESULTS

This section presents sample simulation results to demonstrate the potential of our proposed algorithm. Throughout this section, we assume a 2-ray equal gain Rayleigh fading channel, the total number of subcarriers is 64 for OFDM and the length of the cyclic prefix is larger than the channel delay spread. There are 5 possible transmission modes: No transmission, QPSK with 1 information bit per symbol, 8PSK with 2 information bits per symbol, 16QAM with 3 information bits per symbol, 32QAM with 4 information bits per symbol, and 64QAM with 5 information bits per symbol.

The performance of the proposed single user OFDM-based adaptive TCM in a Rayleigh fading channel is illustrated in Fig. 5, where the desired BER is set to 10^{-4}. The figure shows that by taking advantage of the varying nature of the channel gain, spectral efficiency is significantly improved compared to the fixed rate TCM system with a power gain of 15dB~20dB. Furthermore, the spectral efficiency of adaptive TCM approaches that of an AWGN channel. That is, adapting the coded modulation modes according to instantaneous channel gain can effectively combat the destructive effect of multipath fading. The performance curve of the adaptive TCM system with conservative switching levels, such as used in [7, 9], is also plotted. As is illustrated in the figure, the proposed switching levels outperform the conservative switching levels by around 1dB.

Fig. 6 examines the performance of multiuser OFDM-based adaptive TCM when the desired BER is 10^{-3} and 10^{-4} respectively. Consider a system with 4 users, each with a minimum rate requirement of 32 bits per OFDM symbol. It is clear that when the users share the resources dynamically, spectral and power efficiency can be significantly improved. The figure shows that the multiuser adaptive system considerably outperforms the system with adaptive constellation sizes but fixed multiplexing scheme. The power gain due to the exploitation of multiuser diversity is around 4dB.

VI. CONCLUSIONS

Adaptive transmission has recently been recognized as a potential technique for significantly improving the power efficiency and spectral efficiency when CSI is available at the transmitter. In this paper, we proposed single- and multiuser adaptive TCM algorithms for OFDM transmission. The structure of adaptive trellis makes the family of pragmatic code particularly well suited for adaptive transmission. Transmission modes can be adapted symbol by symbol, instead of codeword by codeword. In our theoretical work, we derived the expressions for BER and BPS for single-user adaptive TCM. A set of appropriate switching levels is then calculated based on these expressions. Our results showed that by exploring the varying nature of the channel, spectral efficiency or power efficiency can be significantly enhanced, and the performance actually approaches that of an AWGN channel. The proposed single user adaptive TCM outperforms fixed rate TCM in a fading channel by 15dB~20dB. It is also shown that in a multiuser environment, "multiuser diversity" can be further explored by dynamic allocation of the bandwidth resources. It is demonstrated that an additional 4 dB gain is achievable compared with adaptive TCM with fixed multiplexing schemes.

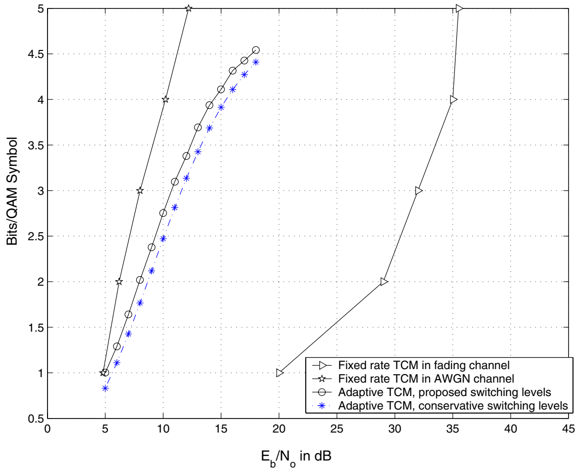

Fig. 5: Performance of single user adaptive pragmatic TCM when target BER is 10^{-4}, compared with that of fixed TCM with the same BER

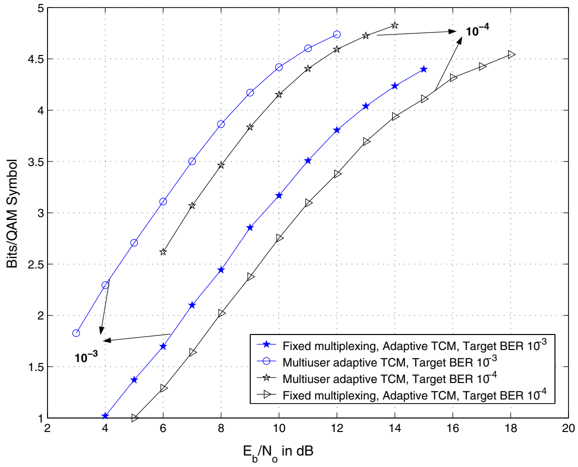

Fig. 6: Performance of multiuser adaptive TCM when target BER is 10^{-3} and 10^{-4} respectively, compared with that of adaptive TCM with fixed multiplexing

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