Power Control for Multi-carrier Communications

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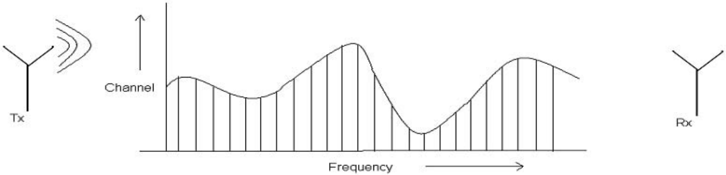
Joint work with R. Berry, M. Honig, M. Chiang, R. Cendrillon, M. Moonen

Sponsors: NSF, Motorola, Alcatel

Yale Seminar, May 2006

Multi-carrier Communication Systems



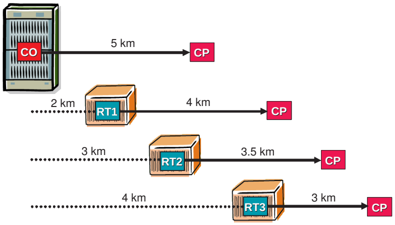
- Split transmit bandwidth into many narrow parallel carriers.
- Robust to ISI (Inter-Symbol-Interference) due to multi-path fading.
- Flexible resource allocation leads to high spectrum efficiency.
- Core technology of: Wi-Fi, WiMAX, DAB/DVB, UWB, DSL, etc.

Resource Allocation in Multi-carrier Systems

- Resource allocation is challenging.
- Simultaneous transmissions over the same carriers lead to interference.
- Optimization problem is typically tightly coupled and non-convex.
- **Objective**: design distributed and optimal resource allocation algorithms to achieve maximum system performance.

Network Models

I: Wireless Ad Hoc Network



II: Wireline DSL Network

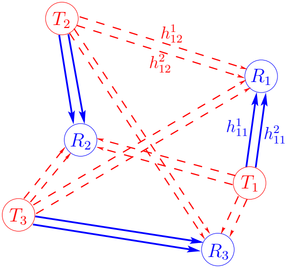
Part I: Wireless Ad Hoc Network

J. Huang, R. Berry and M. Honig, “Distributed Interference Compensation for Wireless Networks,” *IEEE JSAC*, May 2006

Wireless Power Control

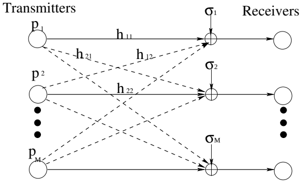
- Well studied in CDMA cellular systems with fixed SINR targets: [Foschini, Miljanic'93], [Yates'95], etc.
- We focus on multi-carrier ad hoc networks and rate adaptive users.
  - One motivation: dynamic spectrum sharing of multiple radio bands.
- Related work: [Chiang'05], [Xi and Yeh'05], etc.
  - Single carrier network.
  - Power control with small stepsizes.

Network Model



- Single-hop transmissions.
- Each user is a transmitter/receiver pair.
- Transmit over several parallel carriers.
    - Interference among users in each carrier.
- First study the special case of single carrier.
- Then consider the generalization for the multi-carrier case.

Single Carrier Model



- A set of $\mathcal{N} = \{1, ..., n\}$ users.
- For each user $n \in \mathcal{N}$:
  - Power constraint: $p_n \in [P_n^{min}, P_n^{max}]$.
  - Received **SINR** (signal-to-interference plus noise ratio):

$$\gamma_n = \frac{h_{n,n} p_n}{\sigma_n + \sum_{m \neq n} h_{n,m} p_m}.$$

  - **Utility function** $U_n(\gamma_n)$: increasing, differentiable, strictly concave.

Network Utility Maximization (NUM) Problem

Problem: 1-SC

$$\max_{\{P_n^{\min} \leq p_n \leq P_n^{\max}, \forall n\}} \sum_n U_n(\gamma_n).$$

- **Technical Challenges**:
    - Coupled across users due to interferences.
    - Could be non-convex in power.
    - Want efficient and distributed algorithm, with limited information exchange and fast convergence.

Benchmark - No Information Exchange

- Each user picks power to maximize its own utility, given current interference and channel gain.
- Results in $p_n = P_n^{max}$ for all n.
  - Can be far from optimal.

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- Each user picks power to maximize its own utility, given current interference and channel gain.
- Results in $p_n = P_n^{max}$ for all n.
  - Can be far from optimal.
- We propose algorithm with limited information exchange.
  - Have nice interpretation as **distributed Pigovian taxation**.
  - Analyze its behavior using **supermodular game theory**.

ADP Algorithm: Asynchronous Distributed Pricing

- **Price Announcing**: user n announces "price" (per unit interference):

$$\pi_n = \left| \frac{\partial U_n(\gamma_n)}{\partial I_n} \right| = \frac{\partial U_n(\gamma_n)}{\partial \gamma_n} \frac{\gamma_n^2}{p_n h_{n,n}}.$$

- **Power Updating**: user n updates power p_n to maximize surplus:

$$S_n = U_n(\gamma_n) - p_n \sum_{m \neq n} \pi_m h_{m,n}.$$

- Repeat two phases asynchronously across users.
- Scalable and distributed: only need to announce single price, and know limited channel gains ($h_{m,n}$).

ADP Algorithm

- Interpretation of prices: **Pigovian taxation**
  - Tax to correct the negative social side-effects of an activity.
  - Improve social welfare.

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- ADP algorithm: distributed discovery of Pigovian taxes
  - When does it converge?
  - What does it converge to?
  - Will it solve Problem **1-SC**?
  - How fast does it converge?

Convergence

- Depends on the utility functions.

Convergence

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- **Coefficient of relative Risk Aversion (CRA)** of $U(\gamma)$:

$$CRA(\gamma) = -\frac{\gamma U''(\gamma)}{U'(\gamma)}.$$

  - larger CRA ⇒ "more concave" U.

Convergence

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- **Coefficient of relative Risk Aversion (CRA)** of $U(\gamma)$:

$$CRA(\gamma) = -\frac{\gamma U''(\gamma)}{U'(\gamma)}.$$

  - larger CRA ⇒ “more concave” U.

- **Theorem**: If for all user n:

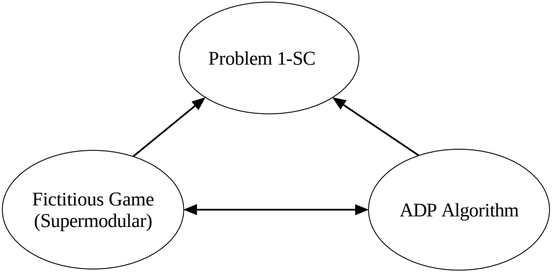
  (a) $P_n^{\min} > 0$, and

  (b) $CRA(\gamma_n) \in [1,2]$ for all feasible γ_n;

  then there is a unique optimal solution of Problem **1-SC**, and the ADP algorithm globally converges to it.

  - E.g. condition (b) is always satisfied with log utilities: $\theta_n \log(\gamma_n)$.
  - Proof: relating this algorithm to a fictitious supermodular game.

Relationship Summary

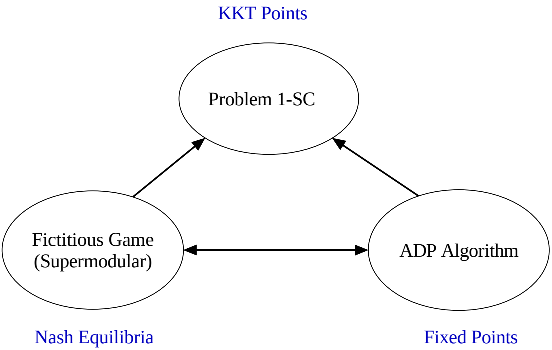


Problem 1-SC

Fictitious Game
(Supermodular)

ADP Algorithm

Relationship Summary



KKT Points

Problem 1-SC

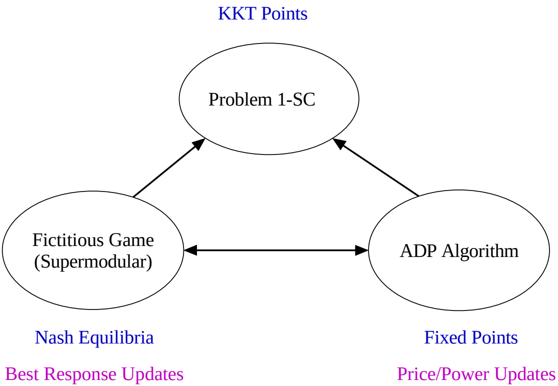
Fictitious Game (Supermodular)

ADP Algorithm

Nash Equilibria

Fixed Points

Relationship Summary



KKT Points

Problem 1-SC

Fictitious Game (Supermodular)

ADP Algorithm

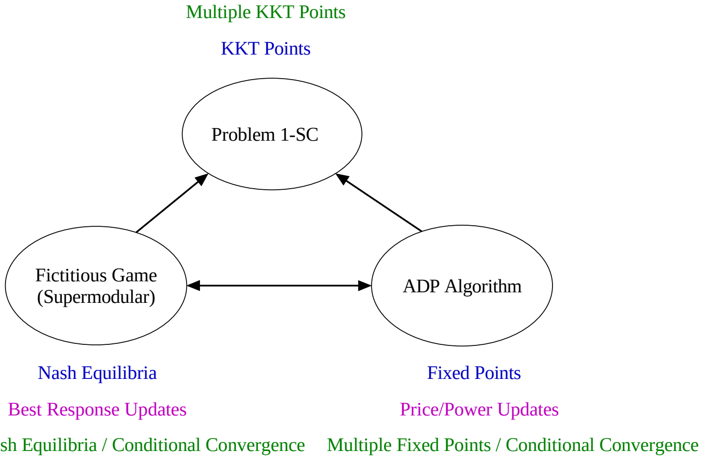
Nash Equilibria

Best Response Updates

Fixed Points

Price/Power Updates

Relationship Summary



Multiple KKT Points

KKT Points

Problem 1-SC

Fictitious Game (Supermodular)

ADP Algorithm

Nash Equilibria

Best Response Updates

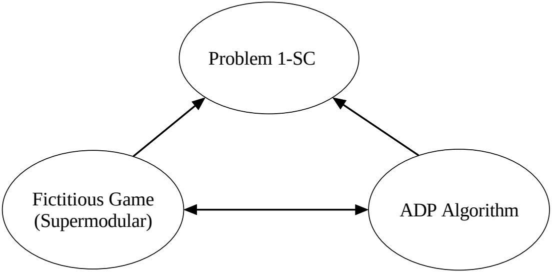
Multiple Nash Equilibria / Conditional Convergence

Fixed Points

Price/Power Updates

Multiple Fixed Points / Conditional Convergence

Relationship Summary



Unique Optimal Solution

Multiple KKT Points

KKT Points

Problem 1-SC

Fictitious Game
(Supermodular)

ADP Algorithm

Nash Equilibria

Best Response Updates

Multiple Nash Equilibria / Conditional Convergence

Unique Nash Equilibrium / Global Convergence

Fixed Points

Price/Power Updates

Multiple Fixed Points / Conditional Convergence

Unique Fixed Point / Global Convergence

Supermodular Games

- A class of games with strategic complementaries
  - Strategy sets are compact subsets of $\mathbb{R}$; and each player's pay-off S_n has increasing differences:

$$\frac{\partial^2 S_n}{\partial x_n \partial x_m} > 0, \forall n, m.$$

- Key properties:

(1) A Nash Equilibrium (N.E.) exists.
(2) If the N.E. is unique, then the asynchronous best response updates will globally converge to it.

Convergence Results

- Construction of the fictitious game
    - Split each user in network into two fictitious players in the game.
    - Choose players' payoffs such that best response updates correspond to the power/price updates in ADP.

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  - Split each user in network into two fictitious players in the game.
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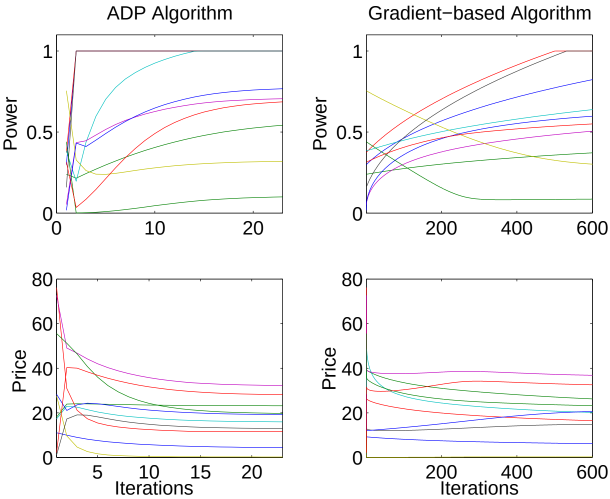
- **Proof**: Condition $CRA(\gamma_n) \in [1,2]$ guarantees
  - The fictitious game is supermodular.
  - The Problem **1-SC** has strictly concave objective function (under log change of variable [Chiang'05]), with convex feasible set.
  - Thus there is a unique global optimal solution/fixed point/N.E.

Convergence Speed



Gradient-based algorithm is based on [Chiang'05], 10 users, log utilities

Multi-carrier Model

Problem: 1-MC

$$\max_{\{\mathbf{p}_n \in \mathcal{P}_n, \forall n\}} \sum_n \sum_k U_n^k(\gamma_n^k).$$

- Assume each user can transmit over K orthogonal channels.
- Received SINR in channel k for user n

$$\gamma_n^k = \frac{h_{n,n}^k p_n^k}{\sigma_n^k + \sum_{m \neq n} h_{n,m}^k p_m^k}$$

- Can allocate power across channels subject to total power constraint:

$$\mathcal{P}_n := \left\{ p_n^k \geq P_n^{\min}, \sum_k p_n^k \leq P_n^{max} \right\}.$$

DADP: Dual ADP Algorithm

- Solves the dual of Problem **1-MC**.

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- **Primal Updates**: solve one subproblem per carrier
  - Under fixed dual prices.
  - Each user n announces π_n^k as before.
  - Each user n chooses p_n^k to maximize

$$S_n^k = U_n^k(\gamma_n^k) - p_n^k \left(\sum_{m \neq n} h_{m,n} \pi_m^k + \mu_n \right).$$

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- **Dual Iterations**: optimize the dual function using subgradient information.

$$\mu_n(t) = \left[\mu_n(t^-) + \kappa \left(\sum_k p_n^k - P_n^{max} \right) \right]^+ .$$

Convergence

- **Theorem**: The DADP algorithm globally and geometrically converges to the unique optimal solution of Problem **1-MC**.
    - Under similar restrictions in single carrier case.
    - With small constant stepsize κ.

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- **Theorem**: The DADP algorithm globally and geometrically converges to the unique optimal solution of Problem **1-MC**.
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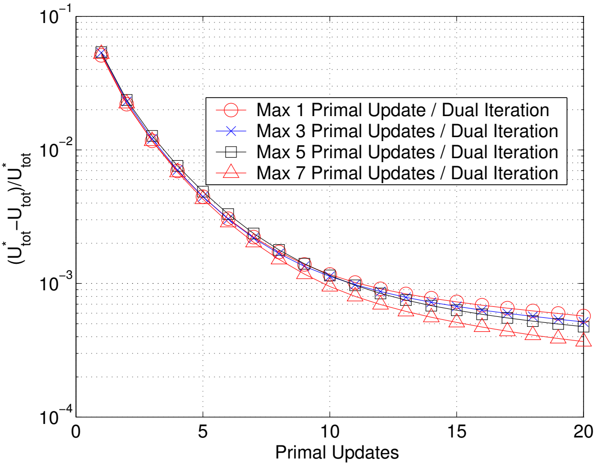
  - With small constant stepsize κ.

- **Proof**:
  - Need to show the Lipschitz continuity and strong convexity of the gradient of dual function.
  - Separation of time-scales assumption: Primal Updates converge between any two adjacent Dual Iterations.

Simulation Results



16 carriers, 50 users, log utilities

Summary

- Consider power control in multi-carrier wireless ad hoc networks.
- Propose **ADP** (Asynchronous Distributed Pricing) algorithms with interpretation of distributed Pigovian taxation.
  - Achieve optimal solution with limited information exchange.
- Supermodular game theory is the key:
  - Convergence of fast power updates without small stepsizes.
  - Analysis of nonconvex problems.
- Extend to multi-carrier case
  - Primal-dual updates.
  - Difference kind of pricing.

Part II: Wireline DSL Network

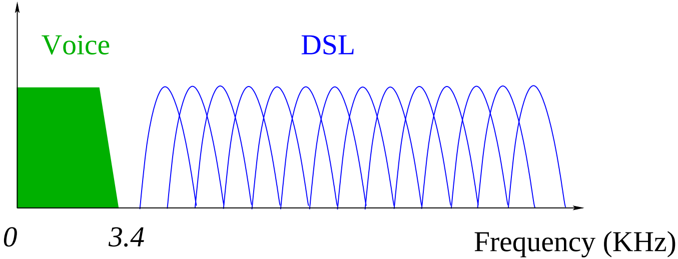
- J. Huang, R. Cendrillon, M. Chiang, and M. Moonen, "Autonomous spectrum balancing (ASB) for digital subscriber lines," to appear in *IEEE ISIT*, July 2006
- R. Cendrillon, J. Huang and M. Chiang, "Autonomous Spectrum Balancing (ASB) for Asynchronous DSL," submitted to *IEEE GlobeCom*, 2006

Wireless vs. DSL

Wireless	DSL
Time Varying Channel	Time-invariant Channel
Mobility	Typical Topology
Need Message Passing	No Message Passing
Pricing	Reference Line

Digital Subscriber Line

- Convert telephone lines into broadband communication media.
- Core technology: **Discrete Multi-Tone** (Cioffi, early 90's).
- Utilize the spectrum unused by voice transmissions and divide into large number of orthogonal carriers/tones.



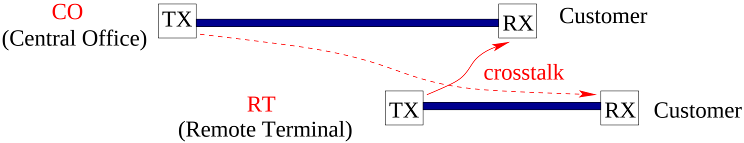
Digital Subscriber Line

- Most ubiquitous and cost-effective access network.
- Current ADSL standard:
  - Utilize up to 1MHz bandwidth.
  - Provide $1.5 - 9$Mbps download speed over a distance of $2.7 - 5.5$Km.
- Provide a variety of services:
  - High-speed Internet access.
  - VoIP.
  - IPTV (AT&T, Jan. 2006).
  - Video-on-demand (AT&T, later 2006 summer)
- A holistic network optimization may significantly improve performance.

FAST Copper Project

- Joint NSF project among Princeton (Chiang), Stanford (Cioffi) and Fraser Research (Fraser).
- Collabration with AT&T.
- Aim at providing DSL broadband service at 100Mbps by joint optimization over Frequency, Amplitude, Space and Time.
- Today focus on the Frequency aspect: **spectrum management**.
  - Performance bottleneck: crosstalks (interferences) among lines.
  - Current practice: static spectrum management.
  - Dynamic spectrum management is needed.

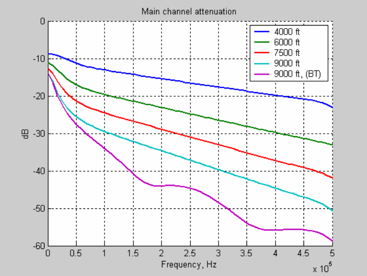
Network Model



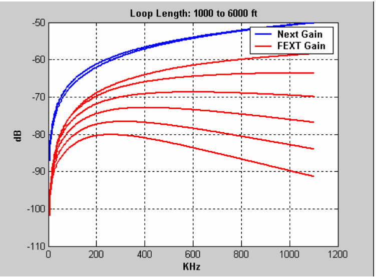
- Each user is a telephone line (transmitter/receiver pair).
- Transmits over multiple carriers/tones.
- Mixed CO/RT deployment
    - Very typical in the United States.
    - CO (central office) can not reach all customers.
    - RT (remote terminal) is deployed to increase DSL footprint.

Channel Characteristics

- Time-invariant channels.
- Topology dependent: channel gain decreases with distance.
- Frequency dependent:
  - Direct channel gain decreases with frequency.
  - Crosstalk channel gain increases with frequency.



Direct Channel Attenuation



Crosstalk Channel Attenuation

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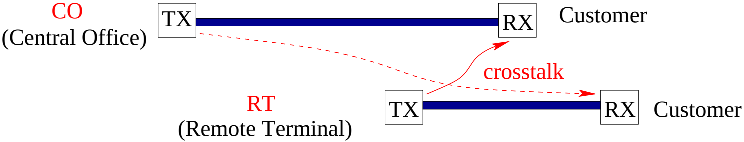
Mixed CO/RT Case



- RT generates strong interference to CO line on high frequencies.
- CO generates little interference to RT line on all frequencies.
- Major performance bottleneck.
- Typical test case for all spectrum management algorithms.

Spectrum Management Problem

- Characterize the Pareto optimal boundary of rate region.

Problem: 2A

$$\begin{aligned} &\underset{\{\mathbf{p}_n \in \mathcal{P}_n\}_n}{\text{maximize}} \; R_1 \\ &\text{subject to } R_n \geq R_n^{\text{target}}, \forall n > 1. \end{aligned}$$

  - User n's achievable rate $R_n = \sum_k \log\left(1 + \frac{p_n^k}{\sum_{m \neq n} \alpha_{n,m}^k p_m^k + \sigma_n^k}\right)$.
  - Total power constraint: $\mathcal{P}_n = \left\{p_n^k \geq 0, \forall k, \sum_k p_n^k \leq P_n^{\max}\right\}$.

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  - Total power constraint: $\mathcal{P}_n = \left\{p_n^k \geq 0, \forall k, \sum_k p_n^k \leq P_n^{\max}\right\}$.

- **Technical Difficulty**:
  - Non-convex and tightly coupled problem.
  - No explicit message passing among users.
  - We want to design distributed, low complexity algorithm with no message passing and near optimal performance.

Dynamic Spectrum Management (DSM)

- State-of-art DSM algorithms:
  - IW: Iterative Water-filling [Yu, Ginis, Cioffi'02]
  - OSB: Optimal Spectrum Balancing [Cendrillon et al.'04]
  - ISB: Iterative Spectrum Balancing [Liu, Yu'05] [Cendrillon, Moonen'05]
  - ASB: Autonomous Spectrum Balancing [Huang et al.'06]

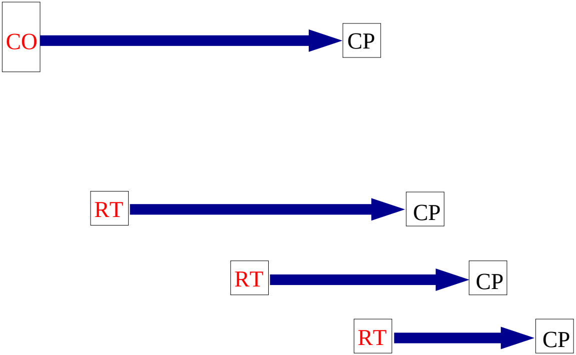
Algorithm	Operation	Complexity	Performance
IW	Autonomous	$O(KN)$	Suboptimal
OSB	Centralized	$O\left(Ke^N\right)$	Optimal
ISB	Centralized	$O\left(KN^2\right)$	Near Optimal
ASB	Autonomous	$O(KN)$	Near Optimal

K: number of carriers N: number of users

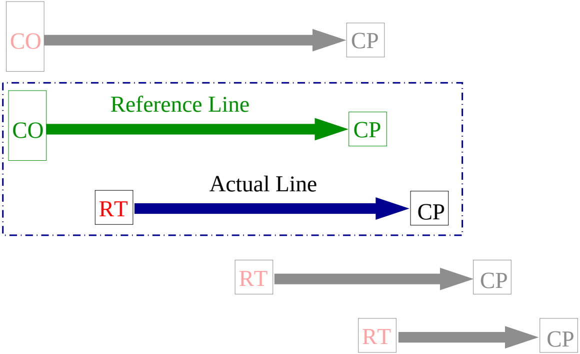
Reference Line Concept

- Provide partial network information.
- **Reference Line**:
  - A virtual line representative of typical CO line in the network.
  - Fixed transmission power and noise on all tones.
  - Fixed crosstalk channels with actual lines.
  - All parameters are obtained through measurement and publicly known.
- Each user maximizes reference line rate, subject to its rate target constraint.
  - Protecting the reference line means protecting the worst victim, thus effectively protecting all other lines.

Reference Line



Reference Line



ASB Algorithm

- Under fixed interferences, each user n solves the following problem:

Problem: 2B

$$\underset{\{\mathbf{p}_n \in \mathcal{P}_n\}}{\text{maximize}} \; R_n^{\text{ref}}$$

$$\text{subject to } R_n \geq R_n^{\text{target}}$$

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where

$$R_n^{\text{ref}} = \sum_k \log\left(1 + \frac{p^{k,\text{ref}}}{\alpha_n^{k,\text{ref}} p_n^k + \sigma^{k,\text{ref}}}\right)$$

  - Only local information is needed.
  - The only interference to the reference line is from user n.
  - User 1's target rate is set to ∞.

- Iterate through users until convergence.

Solve Problem 2B

- Still non-convex, but can be solved by dual decompositions.
  - Duality gap is asymptotically zero with large K [Cendrillon et al.'04].

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- Relax the rate constraint with w_n.

$$\underset{\{\mathbf{p}_n \in \mathcal{P}_n\}}{\text{maximize}} \; R_n^{\text{ref}} + w_n R_n \qquad (\textbf{Step1})$$

  - Adjust w_n to achieve target rate constraint R_n^{target}.

Solve Problem 2B

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    - Adjust w_n to achieve target rate constraint R_n^{target}.
- Relax the total power constraint with λ_n.

$$\underset{\left\{\mathbf{p}_n^k \geq 0\right\}}{\text{maximize}} \; R_n^{k,\text{ref}} + w_n R_n^k - \lambda_n p_n^k, \forall k \qquad \textbf{(Step2)}$$

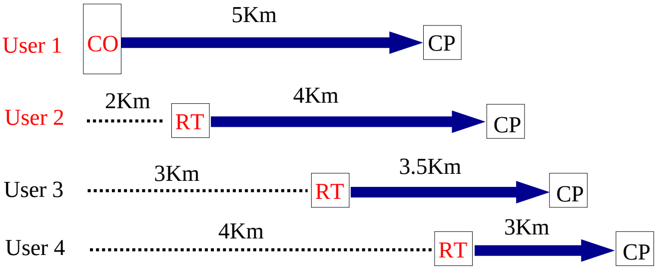
    - Adjust λ_n to achieve $\sum_k p_n^k = P_n^{\max}$.
    - Look at the first order condition and solve a cubic equation.

ASB Algorithm

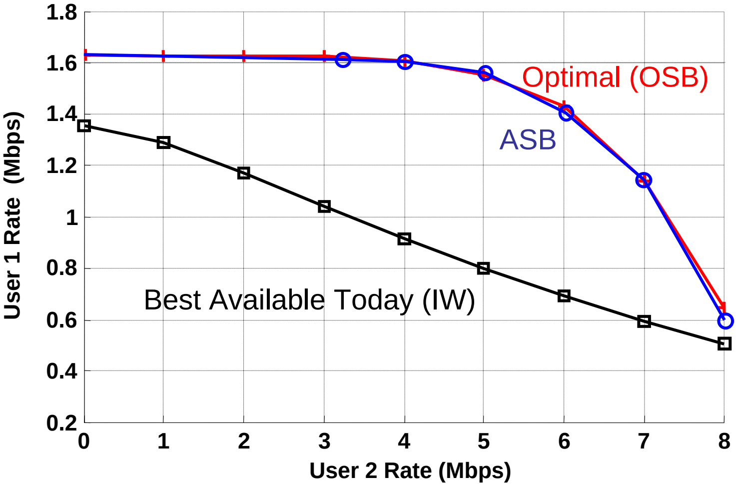
```
repeat
    for each user $n = 1, ..., N$
        repeat
            for each carrier $k = 1, ..., K$, find
                $p_n^k$ = optimal solution of Step2.
            $\lambda_n = \left[\lambda_n + \varepsilon_\lambda \left(\sum_k p_n^k - P_n^{\max}\right)\right]^+$.
            $w_n = \left[w_n - \varepsilon_w \left(\sum_k R_n^k - R_n^{\text{target}}\right)\right]^+$.
        until convergence
    end
until convergence
```

Simulation Setup

- 4 ADSL lines.
- Mixed CO/RT deployment.
- Users 3 and 4 have fixed target rates 2Mbps.
- Find the rate region in terms of users 1 and 2's achievable rates.



Rate Region


Optimal (OSB)
ASB
Best Available Today (IW)
User 1 Rate (Mbps)
User 2 Rate (Mbps)
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
7
8

High SINR Approximation

- Assume reference line operates in high SINR regime on active tones.

$$R_n^{k,\mathrm{ref}} \approx \log\left(\frac{p^{k,\mathrm{ref}}}{\sigma^{k,\mathrm{ref}}}\right) - \frac{\alpha_n^{k,\mathrm{ref}}}{\sigma^{k,\mathrm{ref}}} p_n^k.$$

- User's optimal power is frequency-selective water-filling.

$$p_n^{k*} = \left(\frac{w_n}{\lambda_n + \alpha_n^{k,\mathrm{ref}}/\sigma^{k,\mathrm{ref}}} - \sum_{m \neq n} \alpha_{n,m}^k p_m^k - \sigma_n^k\right)^+ .$$

Convergence of ASB

- **Theorem**: ASB algorithm (under high SINR approximation) globally converges to the unique fixed point if

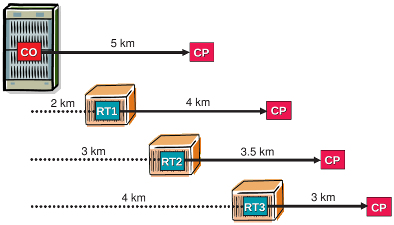
$$\max_{n,m,k} \alpha_{n,m}^{k} < \frac{1}{N-1}.$$

- Independent of the reference line parameters.
- **Proof**: contraction mapping.
    - Recover the convergence of iterative water-filling as a special case.

Summary

- Consider dynamic spectrum management in DSL networks.
- Identify the mixed CO/RT case as the major performance bottleneck.
- Use reference line to represent partial network information.
- Propose ASB (Autonomous Spectrum Balancing) algorithm
  - Fully autonomous.
  - Linear complexity in K and N.
  - Achieves near optimal performance.
- Consider high SINR approximation on the reference line.
  - Close-form optimal solution.
  - Provable convergence.

Summary



Part	**I**	**II**
Motivation	Ad Hoc	DSL
Problem	Nonconvex/Convex	Nonconvex
Algorithm	ADP	ASB
Methodology	Supermodular Game Theory	Reference Line Approximation
Properties	Message Passing	No Message Passing
	Optimal	Near Optimal
	Low Complexity	Low Complexity
	Fast Convergence	Provable Convergence

More publications can be found at
www.princeton.edu/∼jianweih

PADP: Primal ADP Algorithm

- Solve Problem **1-MC** directly.
- **Price Announcing**: user n announces prices $\boldsymbol{\pi}_n = \left(\pi_n^1, ..., \pi_n^K\right)$

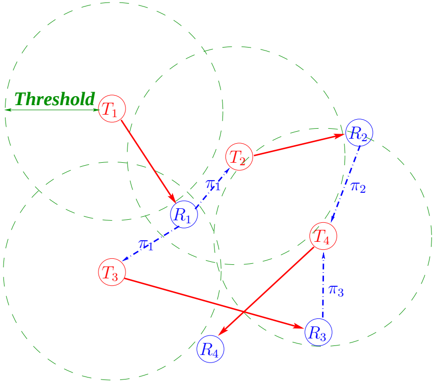
$$\pi_n^k = \left| \frac{\partial U_n^k(\gamma_n^k)}{\partial I_n^k} \right|.$$

- **Power Updating**: user n chooses powers $\mathbf{p}_n = \left(p_n^1, ..., p_n^K\right) \in \mathcal{P}_n$ to maximize surplus

$$S_n = \sum_k \left(U_n^k(\gamma_n^k) - p_n^k \sum_{m \neq n} h_{m,n}^k \pi_m^k \right)$$

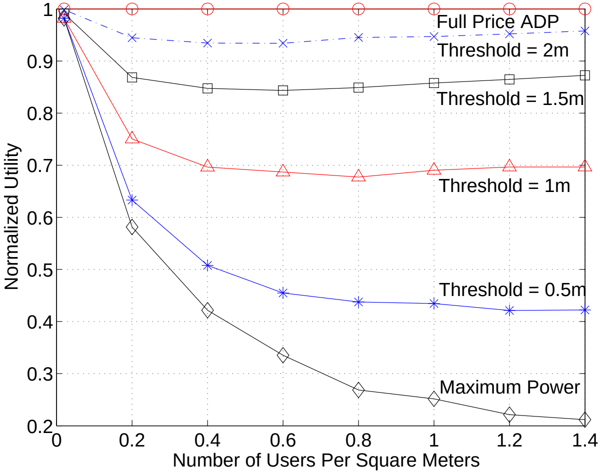
- **Proof** of optimality and convergence:
  - Supermodular game theory.
  - Contraction mapping.

Limited Price Information Exchange



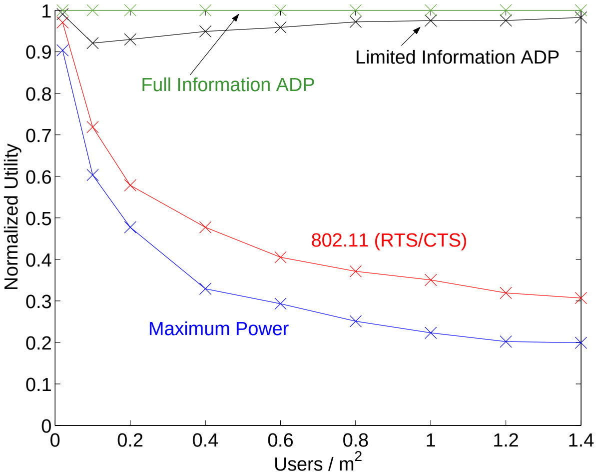
- Previously we assume that each user can decode all the prices.
- In practice, users (receivers) may only decode price messages within threshold distance.

ADP with Limited Pricing



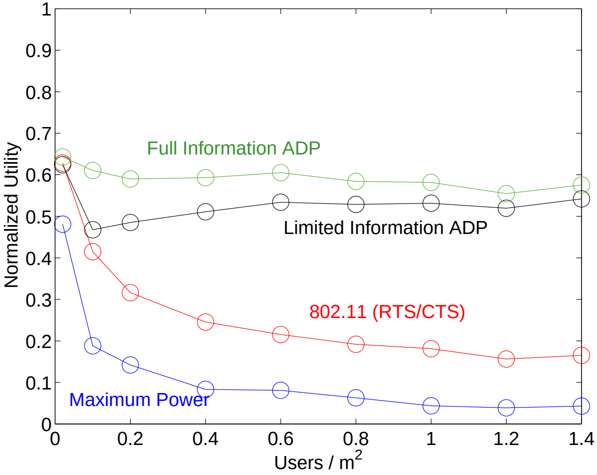
utility $\log(1+\gamma_i)$, 10m × 10m area

Comparison of ADP with 802.11 (RTS/CTS)



rate utility $\log(1+\gamma_i)$, 10m × 10m area

Comparison of ADP with 802.11 (RTS/CTS)



quantized utility $\log(1+\gamma_i)$ ($\{0,5,10,15,20\} bits/Hz$), 10m × 10m area