Resource Allocation in OFDM Systems

Jianwei Huang

Princeton University

Joint work with R. Berry, M. Honig, M. Chiang
V. Subramanian, R. Agrawal, R. Cendrillon, M. Moonen

Sponsors: NSF, Motorola, Alcatel

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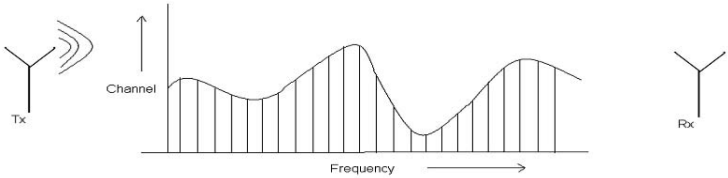
OFDM Systems

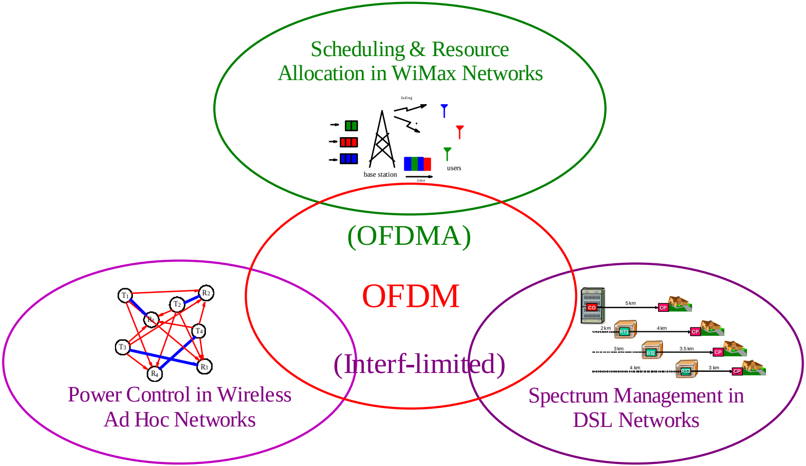


- Frequency band divided into several parallel orthogonal carriers/tones.
- High spectrum efficiency.
- Eliminate inter-symbol-interference (ICI) due to multi-path fading.
- Applications: WiMAX (802.16), Wi-Fi (802.11a/g), DSL, etc.

Resource Allocation in OFDM Systems

- **OFDMA** (Orthogonal Frequency Division Multiple Access) systems
  - Each user is assigned a subset of carriers.
  - No interference among users.
  - Need to determine user-carrier association and power allocation.
- **Interference limited OFDM** systems
  - Each user can transmit over all carriers.
  - Interference among active users in the same carrier.
  - Need to determine power allocation to mitigate interference.

Resource Allocation in OFDM Systems



Model Summary

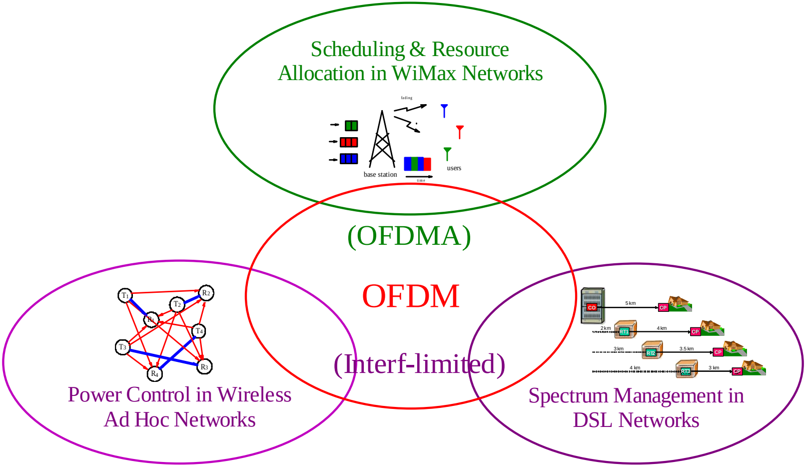
Part	**I**	**II**	**III**
Motivation	WiMAX	Ad Hoc	DSL
Network	Infrastructure	Ad Hoc	Infrastructure
	Wireless	Wireless	Wired
	No Interference	Interference	Interference
Objective	Scheduling & Power Allocation	Power Allocation	Power Allocation

Main References

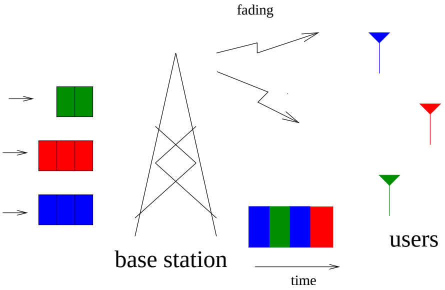
- **WiMAX**: J. Huang, V. Subramanian, R. Agrawal, and R. Berry, "Downlink Scheduling and Resource Allocation for OFDM Systems," *CISS 2006*
- **Ad Hoc**: J. Huang, R. Berry and M. Honig, "Distributed Interference Compensation for Wireless Networks," to appear in *IEEE Journal on Selected Areas in Communications*, May 2006
- **DSL**: J. Huang, R. Cendrillon, M. Chiang, and M. Moonen, "Autonomous spectrum balancing (ASB) for digital subscriber lines," submitted to *ISIT 2006*

More related publications can be found at
www.princeton.edu/∼jianweih

Part I: WiMAX Network



WiMAX Network



- Based on 802.16 and provide MAN broadband connectivity.
  - Cover a distance of up to 5Kms and shared data speed up to 70Mbps.
- Defines a scheduling based MAC protocol.

Gradient-based Scheduling

- We consider downlink channel aware scheduling.
- Many approaches accomplish this via gradient-based scheduling.
  - Assign each user a utility, $U_i(\cdot)$, depending on delay, throughput, etc.
  - Scheduler maximizes choose rate $\mathbf{r} = (r_1, \ldots, r_N)^T$ from the rate region $\mathcal{R}(\mathbf{e})$ to solve:

$$\max_{\mathbf{r} \in \mathcal{R}(\mathbf{e})} \nabla \mathbf{U}(\mathbf{X}(t)) \cdot \mathbf{r} = \max_{\mathbf{r} \in \mathcal{R}(\mathbf{e})} \sum_i w_i r_i,$$

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  - Myopic policy, requires no knowledge of channel or arrival statistics.
  - Depends on the utility functions, could lead to various allocation rules
    - Proportional fair, maximum rate, stabilizing policies, etc.

OFDMA Rate Region

$$\mathcal{R}(\mathbf{e}) = \left\{ \mathbf{r} : \ r_i = \sum_j x_{ij} \log\left(1 + \frac{p_{ij} e_{ij}}{x_{ij}}\right), \ (\mathbf{x}, \mathbf{p}) \in \mathcal{X} \right\},$$

where

- x_{ij} = time fraction of tone j allocated to user i.
- p_{ij} = power allocated to user i on tone j.
- e_{ij} = received SNR/unit power (i.e., channel condition).

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- Feasible region

$$\mathcal{X} := \left\{ (\mathbf{x}, \mathbf{p}) \geq \mathbf{0} : \; x_{ij} \in \{0, 1\}, \; \forall i, j, \quad \sum_i x_{ij} \leq 1, \; \forall j, \; \sum_{ij} p_{ij} \leq P \right\}.$$

Joint Scheduling and Resource Allocation Problem

- Solve at every scheduling interval:

Problem: 1A-INT

$$\max_{(\mathbf{x},\mathbf{p})\in\mathcal{X}} \quad V(\mathbf{x},\mathbf{p}) := \sum_i w_i \sum_j x_{ij} \log\left(1 + \frac{p_{ij} e_{ij}}{x_{ij}}\right)$$

- **Technical Challenges**:
  - Integer constraints on x_{ij}.
  - Want to obtain low-complexity fast algorithms.

Problem with Relaxed Integer Constraints

- We consider the following problem with relaxed integer constraints.

Problem: 1B-RELAX

$$\max_{(\mathbf{x},\mathbf{p})\in\tilde{\mathcal{X}}} \quad V(\mathbf{x},\mathbf{p}) := \sum_i w_i \sum_j x_{ij} \log\left(1 + \frac{p_{ij} e_{ij}}{x_{ij}}\right)$$

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- We will show
  - Typically, optimal solution of Problem **1B-RELAX** is also optimal for Problem **1A-INT**.
  - If not, a near optimal solution of Problem **1A-INT** can be found with almost no additional complexity.

Solve Problem 1B-RELAX

- Convex problem that satisfies Slater's condition.
  - No duality gap.
- Consider **Lagrangian**:

$$L(\mathbf{x}, \mathbf{p}, \lambda, \boldsymbol{\mu}) := \sum_i w_i \sum_j x_{ij} \log\left(1 + \frac{p_{ij} e_{ij}}{x_{ij}}\right) + \lambda\left(P - \sum_{i,j} p_{ij}\right) + \sum_j \mu_j \left(1 - \sum_i x_{ij}\right).$$

- Optimizing over $\mathbf{x}$, $\mathbf{p}$ and $\boldsymbol{\mu}$, find close form solution of $L(\lambda)$.
- Dual function $L(\lambda)$ is convex.
  - Find the optimal λ^* by 1-D iterative search.

Optimal Primal Variables

- Given λ^*, $\boldsymbol{\mu}^*$, let

$$(\mathbf{x}^*, \mathbf{p}^*) = \arg \max_{(\mathbf{x},\mathbf{p}) \in \mathcal{X}} L(\mathbf{x}, \mathbf{p}, \lambda^*, \boldsymbol{\mu}^*).$$

  which lead to

$$x_{ij}^* = \begin{cases} 1, & \mu_{ij}(\lambda^*) = \max_i \mu_{ij}(\lambda^*) \\ 0, & \mu_{ij}(\lambda^*) < \max_i \mu_{ij}(\lambda^*) \end{cases}$$

  - Requires a simple sort of users per tone j.

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  - Requires a simple sort of users per tone j.
- In most cases, no tie occurs on any tone j.
  - Only one user per tone $\Rightarrow$ optimal solution for Problem **1A-INT**.
- Occasionally, ties occur on some tones.
  - Results in multiple users per tone $\Rightarrow$ not primal feasible.
  - Need to break the ties to find a feasible primal solution for Problem **1A-INT**.

Break the Ties for Problem 1A-INT

- Break the ties: choose one user per tone.
  - Each choice corresponds to one power allocation $\mathbf{p}^*$.
- Utilize **subgradient** of dual function $L(\lambda)$: $P - \sum_{ij} p_{ij}^*$.

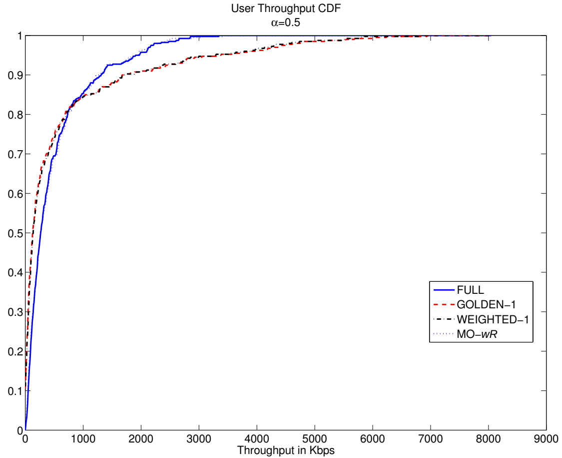
Break the Ties for Problem 1A-INT

- Break the ties: choose one user per tone.
  - Each choice corresponds to one power allocation $\mathbf{p}^*$.
- Utilize **subgradient** of dual function $L(\lambda)$: $P - \sum_{ij} p_{ij}^*$.
- We choose the smallest nonnegative subgradient.
  - This determines a primal feasible tone allocation $\mathbf{x}$.
  - Resulting power constraint might not be tight.
  - Re-optimize the power allocation $\mathbf{p}$.
  - Solve a single-user water-filling solution in finite number of steps (linear in number of tones and users).
- All these add little complexity.

SRT Algorithm: Scheduling and Resource allocation with Tie breaking

1. Consider Problem **1B-RELAX** (i.e., relax the integer constraints).
2. Solve dual function $L(\lambda)$ in close form by optimizing over $\mathbf{x}$, $\mathbf{p}$ and $\boldsymbol{\mu}$.
3. 1-D iterative search for the optimal λ^*.
4. Solve for the primal variables $\mathbf{x}, \mathbf{p}$.
5. If no ties occur, found optimal solution for Problem **1A-INT**. Stop.
6. Break the ties using subgradient information.
7. Re-optimize the power allocation in finite iterations.

User throughput CDFs

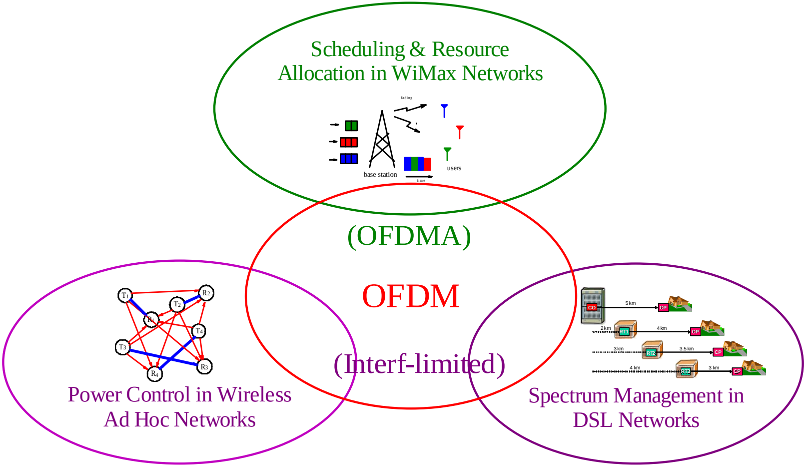


40 users, 5MHz, α fair utility ($\alpha = 0.5$), a total throughput around 20Mbps.

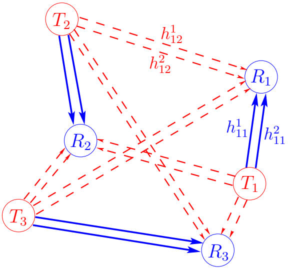
Summary

- Consider mixed integer and convex optimization for joint scheduling and resource allocation in WiMAX network.
- Design SRT algorithm that achieves optimal solution (when no ties occur) with low complexity.
- Breaking the ties adds little complexity.
- Propose simple heuristics with performances close to SRT algorithm.
- **Not covered here**: consider max. SINR constraints and different channelization methods.

Part II: Ad Hoc Network



Multi-Channel Wireless Ad Hoc Network



- Ad hoc network: no fixed infrastructure or centralized controller.
- M users (transmitter-receiver pairs).
- K parallel channels/carriers.
- Interference among users in each channel.
- **Goal**: distributed power control algorithm to maximize network utility.

Special Case: Single Channel Network

Problem: 2A-SC

$$\max_{\{P_n^{\min} \leq p_n \leq P_n^{\max}, \forall n\}} \sum_n U_n(\gamma_n).$$

- Utility $U_n(\gamma_n)$ is increasing and strictly concave in SINR.
- Signal-to-interference plus noise ratio (SINR) of user n

$$\gamma_n = \frac{h_{n,n} p_n}{\sigma_n + \sum_{m \neq n} h_{n,m} p_m}.$$

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- **Technical Challenges**:
  - Coupled across users due to interferences.
  - Could be non-convex in power.
  - Want distributed, low complexity algorithm with fast convergence and optimal performance.

ADP Algorithm: Asynchronous Distributed Pricing

- **Price Announcing**: user n announces "price" (per unit interference):

$$\pi_n = \left| \frac{\partial U_n(\gamma_n)}{\partial I_n} \right| = \frac{\partial U_n(\gamma_n)}{\partial \gamma_n} \frac{\gamma_n^2}{p_n h_{n,n}}.$$

- **Power Updating**: user n updates power p_n to maximize surplus:

$$S_n = U_n(\gamma_n) - p_n \sum_{m \neq n} \pi_m h_{m,n}.$$

- Repeat two phases asynchronously across users.

- Scalable and distributed: only need to announce single price, and know adjacent channel gains ($h_{m,n}$).

ADP Algorithm

- Interpretation of prices: Pigovian taxation
  - Tax for users generating negative externality (interferences) to society.
  - Lead to social optimal solution.

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- ADP algorithm: distributed discovery of Pigovian taxes:
  - Will it converge?
  - What does it converge to?
  - Will it maximize total network utility?
  - How fast does it converge?

Convergence

- **Coefficient of relative Risk Aversion (CRA)** of $U(\gamma)$:

$$CRA(\gamma) = -\frac{\gamma U''(\gamma)}{U'(\gamma)}.$$

  - larger CRA $\Rightarrow$ "more concave" U.

Convergence

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  - larger CRA $\Rightarrow$ "more concave" U.

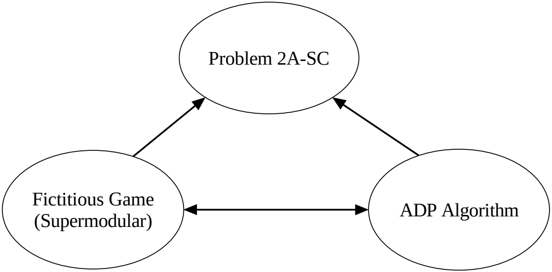
- **Theorem**: If for all user n:

  (a) $P_n^{\min} > 0$, and
  (b) $CRA(\gamma_n) \in [1, 2]$ for all feasible γ_n;

  then there is a unique optimal optimal solution of Problem **2A-SC**, and the ADP algorithm globally converges to it.

  - E.g. condition (b) is always satisfied with log utilities.
  - Proof based on relating this algorithm to a fictitious supermodular game.

Relationship Summary



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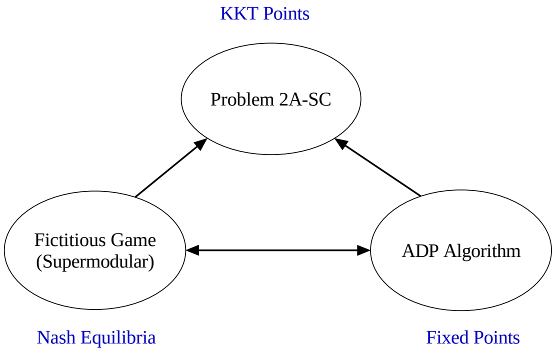

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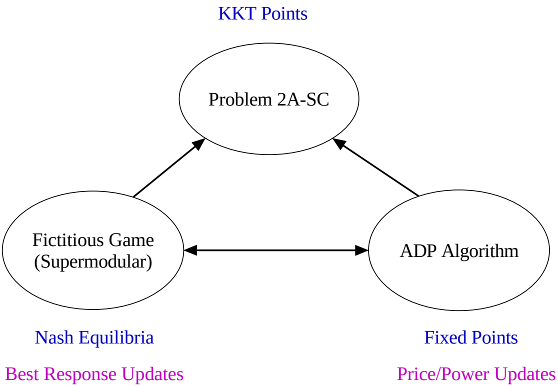

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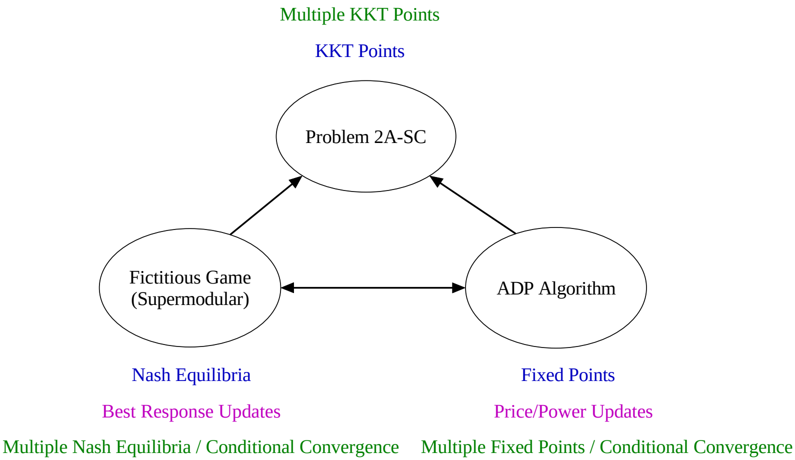

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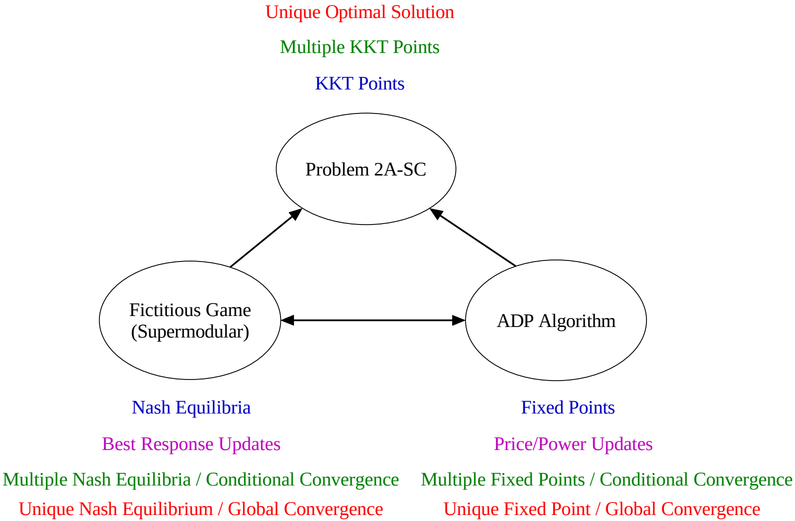

Supermodular Games

- A class of games with "strategic complementaries"
  - Strategy sets are compact subsets of $\mathbb{R}$; and each player's pay-off S_n has "increasing differences":

$$\frac{\partial^2 S_n}{\partial x_n \partial x_m} > 0.$$

- Key properties:
  (1) An N.E. exists.
  (2) If the N.E. is unique, then the asynchronous best response updates will globally converge to it.

Convergence Results

- Construction of the fictitious game
  - Split each user in network into two fictitious players in the game.
  - Choose players' payoffs such that best response updates correspond to the power/price updates in ADP.

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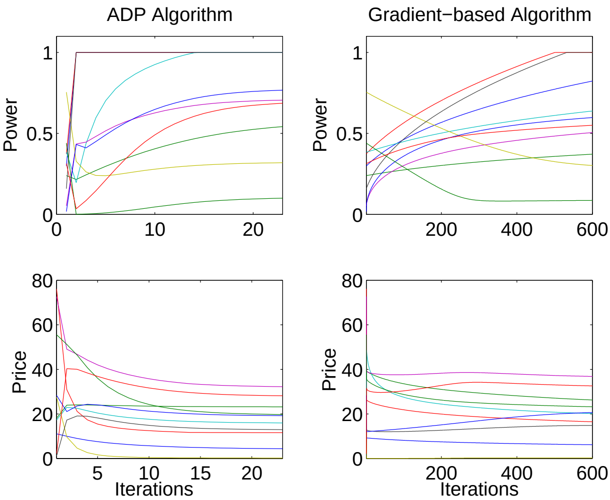
- Proof:
  - Condition $CRA(\gamma_n) \in [1, 2]$ guarantees
    - ★ The fictitious game is supermodular.
    - ★ The Problem **2A-MC** has strictly concave objective function (under log change of variable [Chiang'05]).
  - Thus there is a unique global optimal solution/fixed point/N.E.

Convergence Speed



log utilities, 10 users, Gradient-based Algorithm is based on [Chiang'05]

Multi-channel Model

Problem: 2B-MC

$$\max_{\{\mathbf{p}_n \in \mathcal{P}_n, \forall n\}} \sum_n \sum_k U_n^k(\gamma_n^k).$$

- Assume each user can transmit over K orthogonal channels.
- Received SINR in channel k for user n

$$\gamma_n^k = \frac{h_{n,n}^k p_n^k}{\sigma_n^k + \sum_{m \neq n} h_{n,m}^k p_m^k}$$

- Can allocate power across channels subject to total power constraint:

$$\mathcal{P}_n := \left\{ p_n^k \geq P_n^{\min}, \sum_k p_n^k \leq P_n^{max} \right\}.$$

Dual ADP (DADP) Algorithm

- Two classes of prices:
  - User still announces an **interference price** π_n^k on each channel k.
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  - User chooses power p_n^k to maximize:

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  - Interference price π_n^k updated as in single channel ADP algorithm.
- **Dual Iteration**: resource price follows subgradient updates:

$$\mu_n(t) = \left[\mu_n(t^-) + \kappa \left(\sum_k p_n^k - P_n^{max} \right) \right]^+.$$

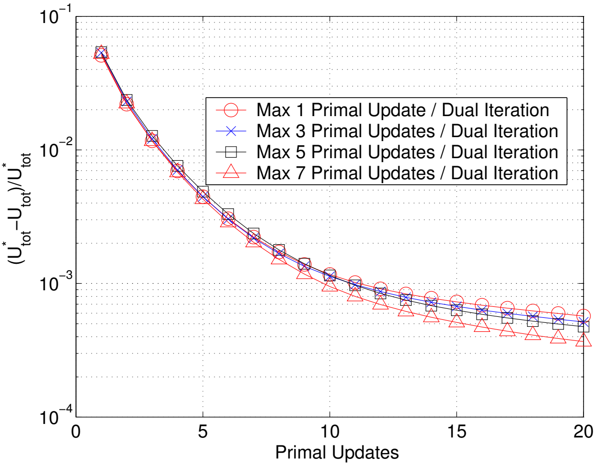
Convergence

- **Theorem**: The DADP algorithm globally and geometrically converges to the unique optimal solution of Problem **2B-MC**.
  - Under similar restrictions on the utility functions in single channel case.
  - With small constant stepsize κ.

Convergence

- **Theorem**: The DADP algorithm globally and geometrically converges to the unique optimal solution of Problem **2B-MC**.
  - Under similar restrictions on the utility functions in single channel case.
  - With small constant stepsize κ.
- **Proof**:
  - Need to show the Lipschitz condition and strong convexity of the gradient of dual function.
  - Separation of time-scales assumption: Primal Updates converges between any two adjacent Dual Iterations

Simulation Results

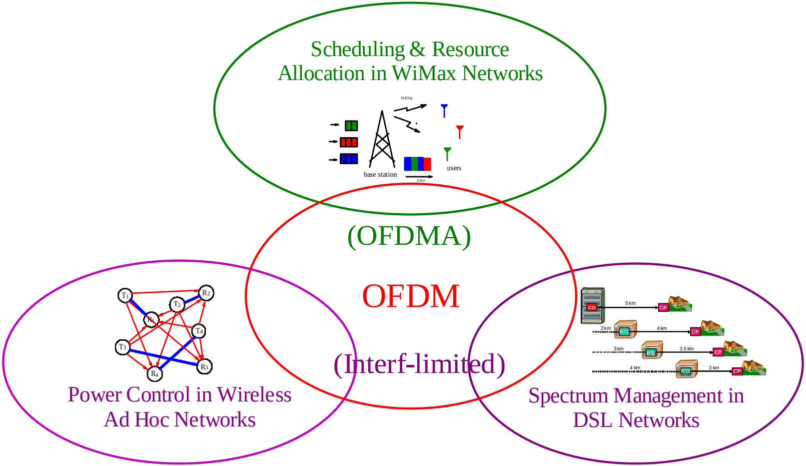


log utilities, 16 channels, 50 users

Summary

- Consider power control in multi-channel wireless ad hoc networks.
- Propose dual-based ADP algorithm that achieves optimal solution with fast convergence (under proper conditions of the utility functions).
- **Not covered here**:
  - Primal-based ADP algorithm to solve both convex and non-convex multi-channel power control problem.
  - Convergence of ADP algorithm when each user is limited to choose one out of many channels.

Part III: DSL Network



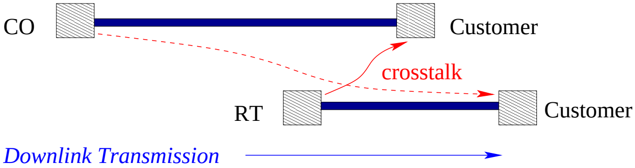
Digital Subscriber Line

- **Discrete Multi-Tone** introduced in early 90's by John Cioffi.
- Utilize the spectrum unused by voice transmissions and divide into large number of subchannels.
- Convert traditional telephone twisted-pair copper wires to broadband communication media.

Current Efforts

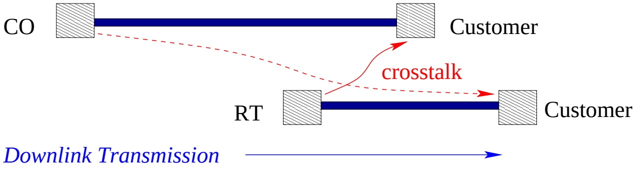
- Performance bottleneck: crosstalks (interferences) among lines.
  - Current practice: static spectrum management.
  - A new wave of **dynamic spectrum management** since 2002.
- FAST Copper project:
  - Joint NSF project among Princeton, Stanford and Fraser Research.
  - Collabration with AT&T.
  - Aim at providing DSL broadband service up to 100Mbps by joint optimization over Frequency, Amplitude, Space and Time.
  - Today we will focus on the Frequency aspect.

Multiple-line Channel Model



- Mathematically similar as the multi-channel wireless ad hoc network.
  - Difference: channel can be considered as time-invariant.
- Crosstalks are highly distance and frequency dependent:
  - Decrease with distance.
  - Increase with frequency.

Multiple-line Channel Model



- Mathematically similar as the multi-channel wireless ad hoc network.
  - Difference: channel can be considered as time-invariant.
- Crosstalks are highly distance and frequency dependent:
  - Decrease with distance.
  - Increase with frequency.
- In the CO/RT mixed case
  - RT generates excessive interference to the CO line.
  - CO generates little interference to the RT line.
  - Also called near-far problem: performance bottleneck in the US.

Spectrum Management Problem

Problem: 3

$$\underset{\{p_n^k\}_{k,n}}{\text{maximize}} \sum_n w_n R_n$$

$$\text{subject to } R_n = \sum_k \log\left(1 + \frac{p_n^k}{\sum_{m \neq n} \alpha_{n,m}^k p_m^k + \sigma_n^k}\right), \forall n$$

$$\sum_k p_n^k \leq P_n^{\max}, \forall n$$

$$p_n^k \geq 0, \forall k, n.$$

- **Technical Difficulty**:
  - Highly non-convex and tightly coupled problem.
  - No explicit message passing among users is desired.
  - Want find distributed and low complexity algorithm with near optimal performance.

Dynamic Spectrum Management (DSM)

- State-of-art of dynamic spectrum management
  - Iterative Water-filling (IW) [Yu, Ginis, Cioffi'02]
  - Optimal Spectrum Balancing (OSB) [Cendrillon et al.'04]
  - Iterative Spectrum Balancing (ISB) [Cendrillon, Moonen'05] [Liu, Yu'05]
  - Autonomous Spectrum Balancing (ASB) [Huang et al.'06]

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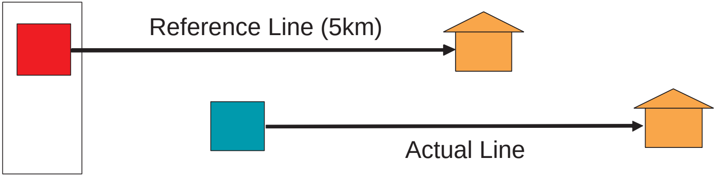
Algorithm	Operation	Complexity	Performance
IW	Autonomous	$O(KN)$	Suboptimal
OSB	Centralized	$O\left(Ke^N\right)$	Asymptotic Optimal
ISB	Centralized	$O\left(KN^2\right)$	Near optimal
ASB	Autonomous	$O(KN)$	Near optimal

Autonomous Spectrum Balancing

- Finding the optimal solution requires global crosstalk information.
- In mixed CO/RT case, need to protect the service on the CO line.

Autonomous Spectrum Balancing

- Finding the optimal solution requires global crosstalk information.
- In mixed CO/RT case, need to protect the service on the CO line.
- **Reference Line**:
  - A fictitious line representative of typical CO line.
  - All parameters are fixed and known to all users.
- Each user tries to maximize a weighted sum of its own rate and reference line's rate.



ASB Algorithm

- Each user n solves the following problem

$$\begin{aligned} &\underset{\{p_n^k\}_k \geq 0}{\text{maximize}} \; w_n \sum_k \left(1 + \frac{p_n^k}{\sum_{m \neq n} \alpha_{n,m}^k p_m^k + \sigma_n^k}\right) + \sum_k \log\left(1 + \frac{\tilde{p}^k}{\tilde{\alpha}_n^k p_n^k + \tilde{\sigma}^k}\right) \\ &\text{subject to} \sum_k p_n^k \leq P_n^{\max}, \end{aligned}$$

- Iterate through users until convergence.

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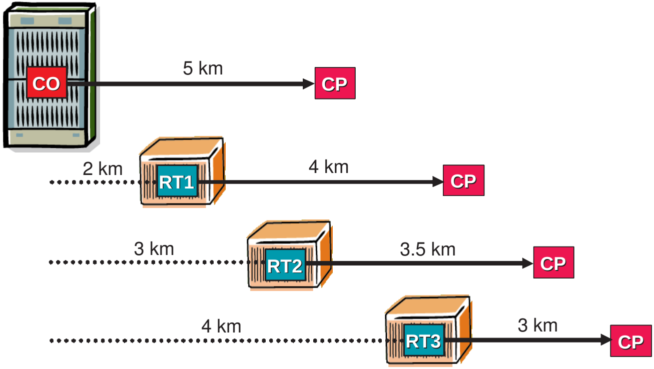
$$\text{subject to} \sum_k p_n^k \leq P_n^{\max},$$

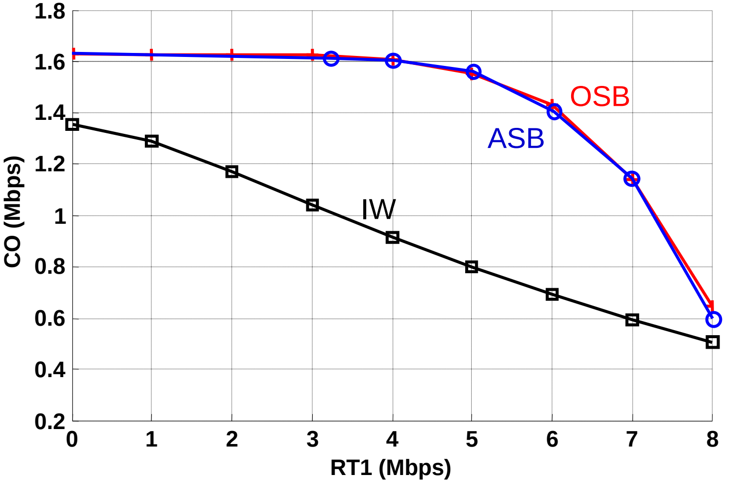
- Iterate through users until convergence.
- Only need local information.
- Still non-convex, but can be solved by dual decomposition.
  - Duality gap is asymptotically zero when the number of tones is high [Cendrillon et al.'04].
  - Per tone subproblem: find optimal solution by solving a cubic equation.

Simulation Setup

- 4 ADSL lines
- Mixed CO/RT deployment
- Target rate set to 2Mbps on RT2 and RT3



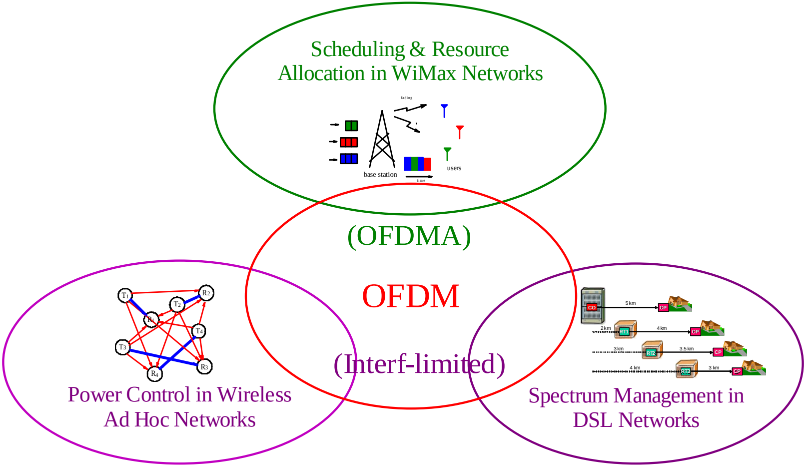
Performance



Summary

- Consider dynamic spectrum management in DSL networks.
- Propose ASB algorithm, which is fully autonomous, with low complexity, and achieves near optimal performance.
- **Not covered here**:
  - High SINR approximation on the reference line to with even lower complexity and faster convergence.
  - Consider asynchronous transmissions, where the channels are not orthogonal to each other.

Conclusions



Results Summary

Part	I	II	III
Motivation	WiMAX	Ad Hoc	DSL
Problem	Integer/Convex	Nonconvex/Convex	Nonconvex
Methodology	Integer Relaxation Tie Breaking	Supermodular Game Theory	Reference Line Approximation
Algorithm	SRT	ADP	ASB
Properties	Centralized	Distributed	Distributed
	(Near) Optimal	Optimal	Near Optimal
	Low Complexity	Low Complexity	Low Complexity
	Converges	Fast Convergence	Converges in pratice