Autonomous Spectrum Balancing (ASB) for Frequency Selective Interference Channels

Jianwei Huang

Princeton University

Joint work with Raphael Cendrillon, Mung Chiang, Marc Moonen

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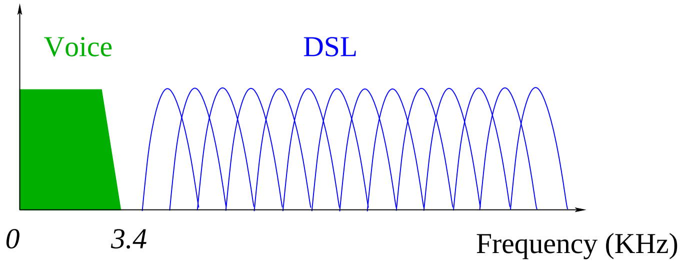
Digital Subscriber Line (DSL) Channels

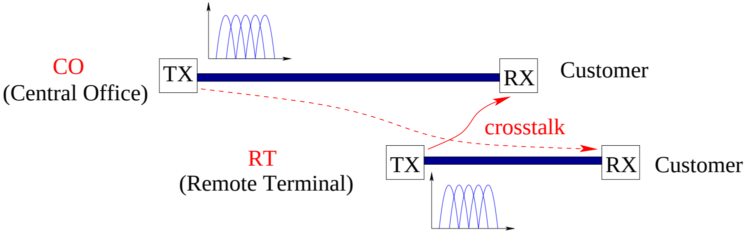
- Wireline channels based on traditional telephone networks.
- **Multi-user frequency selective interference channel**:
  - Multi-carrier transmissions over large bandwidth.
  - Static and frequency selective channel responses.
  - Multiple telephone lines interfere with each other.



Digital Subscriber Line (DSL) Channels

- Most ubiquitous and cost-effective access network.
- Provide broadband communication capability:
  - Current ADSL: 9Mbps download speed over 2.7Km and 1MHz.
  - Applications: VoIP, IPTV, Video-on-demand, ...
- FAST Copper:
  - Joint NSF project among Princeton, Stanford and Fraser Research.
  - Collaboration with AT&T: $4 billion fiber/DSL deployment
  - Aim at providing DSL broadband service at 100Mbps by joint optimization over Frequency, Amplitude, Space and Time.
  - Today focus on the Frequency aspect: **spectrum management** (power control).

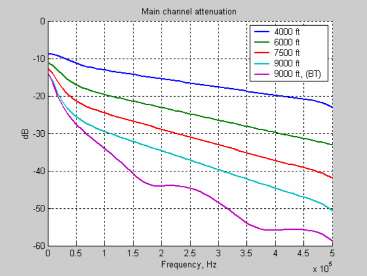
Channel Model



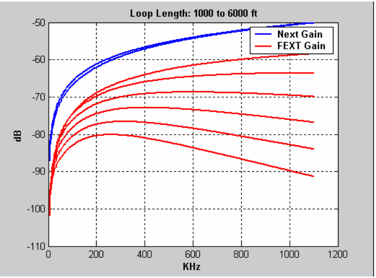
- A typical DSL channel:
  - Each line/user is transmitter-receiver pair.
  - Transmits over multiple carriers/tones.
  - Generate mutual interferences/crosstalks.
  - RT extends the reach of CO and increases the DSL footprint.

Channel Characteristics

- Static (time-invariant) channels.
- Topology dependent: channel gain decreases with distance.
- Frequency dependent:
  - Direct channel gain decreases with frequency.
  - Crosstalk channel gain increases with frequency.

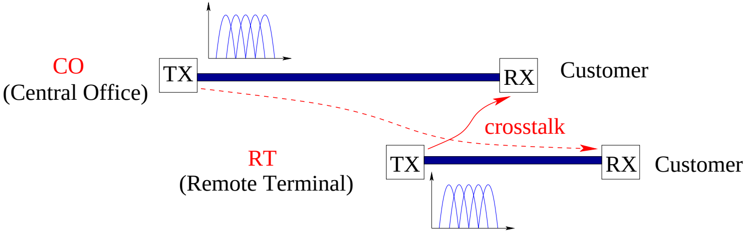


Direct Channel Attenuation



Crosstalk Channel Attenuation

Mixed CO/RT Case



- RT generates strong interference to CO line on high frequencies.
- CO generates little interference to RT line on all frequencies.
- Major performance bottleneck.
- Typical test case for all spectrum management algorithms.

Spectrum Management Problem

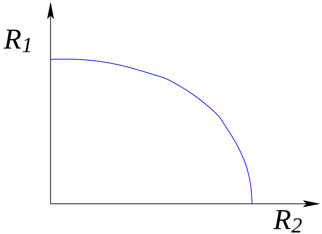
- Characterize the Pareto optimal boundary of rate region [Centrillon et al.'04].

Problem A

$$\underset{\{\mathbf{p}_n \in \mathcal{P}_n\}_n}{\text{maximize}} \; R_1$$

$$\text{subject to } R_n \geq R_n^{\text{target}}, \forall n > 1.$$

  - User n's achievable rate $R_n = \sum_k \log \left(1 + \frac{p_n^k}{\sum_{m \neq n} \alpha_{n,m}^k p_m^k + \sigma_n^k} \right)$.
  - Total power constraint: $\mathcal{P}_n = \left\{ p_n^k \geq 0, \forall k, \sum_k p_n^k \leq P_n^{\max} \right\}$.



Difficulties of Solving Problem A

- Non-convex in power.
- Tightly coupled across users & tones.
- No centralized controller.
- No explicit message passing among users.
- **Objective**: design an algorithm that is
  - Low complexity.
  - Distributed.
  - With no message passing.
  - Achieve near optimal performance.

Dynamic Spectrum Management (DSM)

- State-of-art DSM algorithms:
  - IW: Iterative Water-filling [Yu, Ginis, Cioffi'02]
  - OSB: Optimal Spectrum Balancing [Cendrillon et al.'04]
  - ISB: Iterative Spectrum Balancing [Liu, Yu'05] [Cendrillon, Moonen'05]
  - ASB: Autonomous Spectrum Balancing [Huang et al.'06]

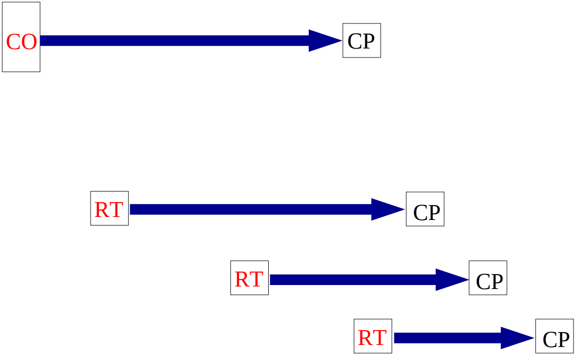
Algorithm	Operation	Complexity	Performance
IW	Autonomous	$O(KN)$	Suboptimal
OSB	Centralized	$O(Ke^N)$	Optimal
ISB	Centralized	$O(KN^2)$	Near Optimal
ASB	Autonomous	$O(KN)$	Near Optimal

K: number of carriers N: number of users

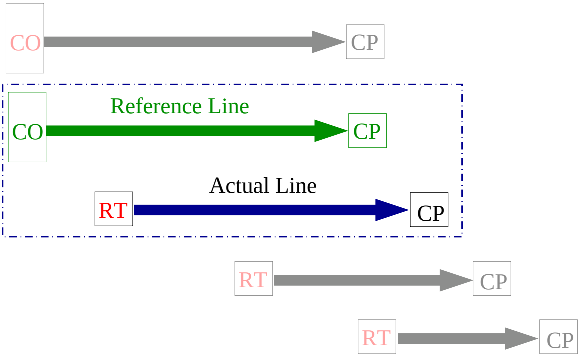
Key Idea

- What we know
  - Selfishness is detrimental.
  - Centralized computation is infeasible.
  - Dynamic pricing among users requires explicit information exchange.
- Our solution: **reference line**: static pricing for static channel
  - A virtual line representative of a worst victim in the network.
  - Parameters (power, noise, crosstalk) are fixed and publicly known.
  - Already available in the current industry standards through long term field measurements, but not used in the right way.
- Each user tries to protect the reference line and avoid pure selfish behavior.

Reference Line



Reference Line



ASB Algorithm

- User n solves the following problem:

Problem B

$$\underset{\{\mathbf{p}_n \in \mathcal{P}_n\}}{\text{maximize}} \; R_n^{\text{ref}}$$
$$\text{subject to } R_n \geq R_n^{\text{target}}$$

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where

$$R_n^{\text{ref}} = \sum_k \log\left(1 + \frac{p^{k,\text{ref}}}{\alpha_n^{k,\text{ref}} p_n^k + \sigma^{k,\text{ref}}}\right)$$

  - Only local information is needed.
  - The only interference to the reference line is from user n.

- Iteration across users until power spectrum density (PSD) converge.

Solve Problem B

- Still non-convex, but can be solved by dual decompositions.
  - Duality gap is asymptotically zero with large K [Cendrillon et al.'04].

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- Relax the rate constraint with w_n:

$$\underset{\{\mathbf{p}_n \in \mathcal{P}_n\}}{\text{maximize}} \; R_n^{\text{ref}} + w_n R_n \qquad \textbf{(Step1)}$$

- Relax the total power constraint with λ_n:

$$\underset{\left\{\mathbf{p}_n^k \geq 0\right\}}{\text{maximize}} \; R_n^{k,\text{ref}} + w_n R_n^k - \lambda_n p_n^k, \forall k \qquad \textbf{(Step2)}$$

  - How to solve: examine the first order condition and solve a cubic equation.

ASB Algorithm

repeat

for each user $n = 1, ..., N$

repeat

for each carrier $k = 1, ..., K$, find

p_n^k = optimal solution of **(Step2)**.

$$\lambda_n = \left[\lambda_n + \varepsilon_\lambda \left(\sum_k p_n^k - P_n^{\max}\right)\right]^+.$$

$$w_n = \left[w_n - \varepsilon_w \left(\sum_k R_n^k - R_n^{\text{target}}\right)\right]^+.$$

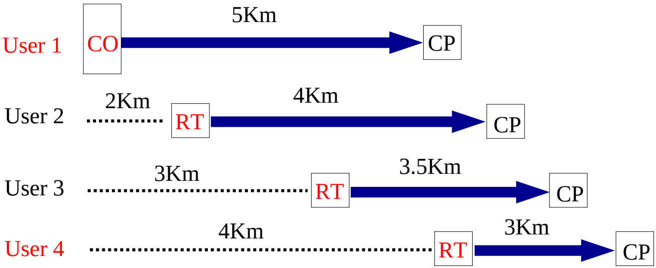
until convergence

end

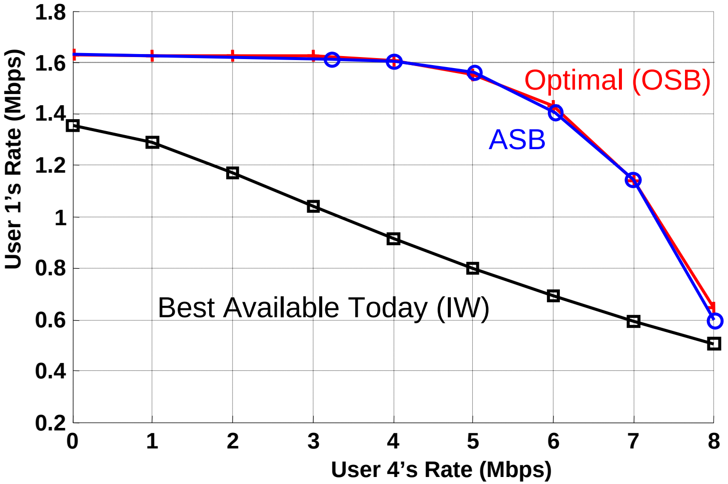
until convergence

Simulation Setup

- 4 ADSL lines.
- Mixed CO/RT deployment.
- Users 2 and 3 have fixed target rates 2Mbps.
- Find the rate region in terms of users 1 and 4's achievable rates.

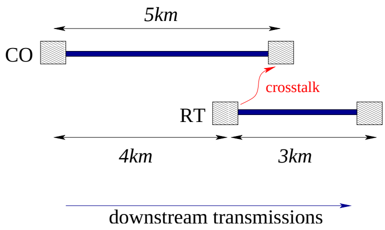


Rate Region

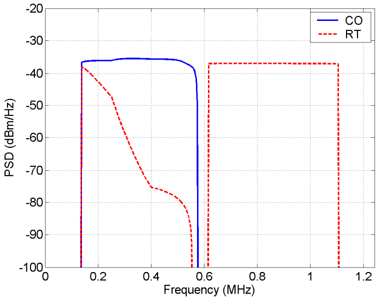


1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
User 1's Rate (Mbps)
0
1
2
3
4
5
6
7
8
User 4's Rate (Mbps)
Optimal (OSB)
ASB
Best Available Today (IW)

Power Allocation



Two-line Topology



Power Allocation

Connection with Iterative Water-filling

- Assume reference line operates in high SINR regime on active tones.

$$R_n^{k,\text{ref}} \approx \log\left(\frac{p^{k,\text{ref}}}{\sigma^{k,\text{ref}}}\right) - \frac{\alpha_n^{k,\text{ref}}}{\sigma^{k,\text{ref}}} p_n^k.$$

- Actual user n's optimal power is frequency-selective water-filling.

$$p_n^{k*} = \left(\frac{w_n}{\lambda_n + \alpha_n^{k,\text{ref}}/\sigma^{k,\text{ref}}} - \sum_{m\neq n} \alpha_{n,m}^k p_m^k - \sigma_n^k\right)^+ .$$

  - Includes traditional iterative water-filling as a special case (i.e., no reference line).

Convergence of ASB

- **Theorem**: ASB algorithm (under high SINR approximation) globally converges to the unique fixed point under both sequential and parallel updates if the crosstalk channels among actual users satisfy:

$$\max_{n,m,k} \alpha_{n,m}^{k} < \frac{1}{N-1}.$$

  - Independent of the reference line parameters.
  - Recover the convergence of iterative water-filling as a special case.

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  - Recover the convergence of iterative water-filling as a special case.

- **Proof**: contraction mapping.

Proof of Convergence

- **Key Lemma**: min-max of an increasing function and an decreasing function is achieved at the intersection.
- Construct two such functions based on the ASB algorithm.
- Show the maximum difference between the PSD during adjacent iterations is decreasing.

$$\max_n \max \left\{ \sum_k \left[p_k^{n,t+1} - p_k^{n,t} \right]^+ , \sum_k \left[p_k^{n,t+1} - p_k^{n,t} \right]^- \right\}$$
$$\leq \max_n \max \left\{ \sum_k \left[p_k^{n,t} - p_k^{n,t-1} \right]^+ , \sum_k \left[p_k^{n,t} - p_k^{n,t-1} \right]^- \right\}$$

  - Sequential updates: bound the maximum eigenvalue of the mapping matrix (motivated by [Chung'03]).
  - Parallel updates: more realistic with cleaner proof (motivated by [Yu'02]).

Summary

- **Topic**: spectrum management in DSL multiuser interference channels.
- **Key idea**: static pricing using reference line.
- **Algorithm**: ASB: autonomous and low complexity.
- **Performance**: close to optimal, provable convergence.
- **Related and future work:**
  - ASB for asynchronous transmissions.
  - Joint statistical multiplexing, scheduling and spectrum management.