

Performance of Distributed Utility-Based Power Control for Wireless Ad Hoc Networks

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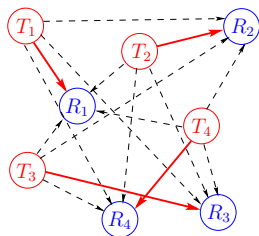
MILCOM 2005

Power Control



- Power control useful for mitigating interference and improving wireless network performance.
- Well studied in CDMA cellular systems (e.g. Yates, Hanly).
 - ▶ Focused on cellular model with fixed SINR targets/user.
- Here we focus on “ad hoc” environments and rate adaptive users.
 - ▶ One motivation: “spectrum sharing” in unlicensed/open spectrum.

Spectrum Sharing

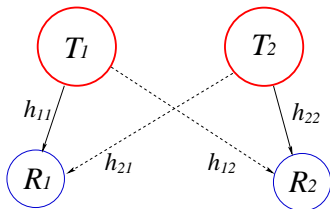


- Focus on sharing a *single* band of spectrum in a given geographic area among competing *users*.
 - ▶ can view as an ad hoc network
- Each “user” is a single transmitter/receiver pair.
- Want to utilize spectrum *efficiently*.
- Want to accomplish this via a distributed algorithm with limited information exchange (*scalability*).
 - ▶ No central controller or spectrum manager.

Talk Outline

- Network model and performance metric.
- Price-based power control algorithm: optimality and convergence.
- Numerical study of performance.

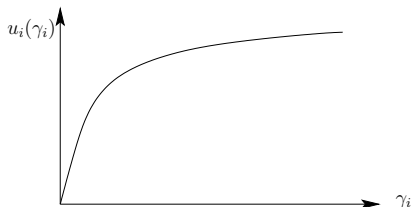
Network Model



- Static channels.
- Each user spreads signal over entire bandwidth of B Hz.
- User i transmits with power $p_i \in [P_i^{min}, P_i^{max}]$.
- User i 's QoS depends on received SINR,

$$\gamma_i = \frac{h_{ii} p_i}{n_0 + \frac{1}{B} \sum_{j \neq i} h_{ji} p_j}$$

User Preferences



- All users are rate-adaptive with *elastic* demands.
- User's QoS preferences given by utility $u_i(\gamma_i)$.
 - ▶ Increasing, twice differentiable, strictly concave function of γ_i .
- **Goal:** determine power allocation that maximizes total utility:

$$\sum_i u_i(\gamma_i)$$

Benchmark - No Information Exchange

- Each user chooses power to maximize its own utility given the interference and channel gain.
- Results in $p_i = P_i^{max}$ for all i .
 - ▶ Can be far from optimal.
- We present an algorithm with limited information exchange that improves upon this.

Asynchronous Distributed Pricing (ADP) Algorithm

- **Price Announcement:** each user i announces a “price”:

$$\pi_i = \left| \frac{\partial u_i(\gamma_i)}{\partial \left(\sum_{j \neq i} p_j h_{ji} \right)} \right| = \frac{\partial u_i(\gamma_i)}{\partial \gamma_i} \frac{\gamma_i^2}{B p_i h_{ii}}.$$

- **Power Update:** user i updates power p_i to maximize surplus:

$$s_i = \underbrace{U_i(\gamma_i(p_i))}_{\text{Utility}} - \underbrace{p_i \sum_{j \neq i} \pi_j h_{ij}}_{\text{Payment}}$$

- Repeat these steps asynchronously.
- Only need to announce a **single** price and know “**adjacent**” channel gains (h_{ij}).

ADP Algorithm

- Interpretation of interference price: **Pigovian taxation**
 - ▶ Users pay taxes on transmission power (which generates negative externalities to other users)
 - ▶ Typically imposed by a central manager (e.g., government), which knows global information.

ADP Algorithm

- Interpretation of interference price: **Pigovian taxation**
 - ▶ Users pay taxes on transmission power (which generates negative externalities to other users)
 - ▶ Typically imposed by a central manager (e.g., government), which knows global information.
- ADP algorithm: distributed discovery of Pigovian tax:
 - ▶ When does this converge?
 - ▶ If it converges what is the resulting allocation?
 - ▶ When is the allocation (socially) optimal, i.e., maximizes the total utility, $\sum_i u_i(\gamma_i)$?

Convergence

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- Define the *coefficient of relative risk aversion* for utility $U_i(\gamma_i)$ as

$$G_i(\gamma_i) := -\frac{\gamma_i U_i''(\gamma_i)}{U_i'(\gamma_i)}.$$

- ▶ larger $G_i(\gamma_i) \Rightarrow$ “more concave” U_i

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- **Prop:** If for all i
 - (1) $P_i^{\min} > 0$, and
 - (2) $G_i(\gamma_i) \in [a, b] \subset [1, 2]$ for all feasible γ_i ;

then there is a unique optimal power allocation and the ADP algorithm globally converges to it.

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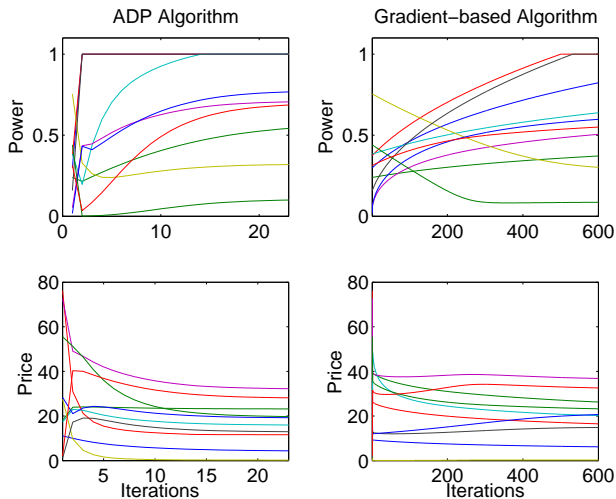
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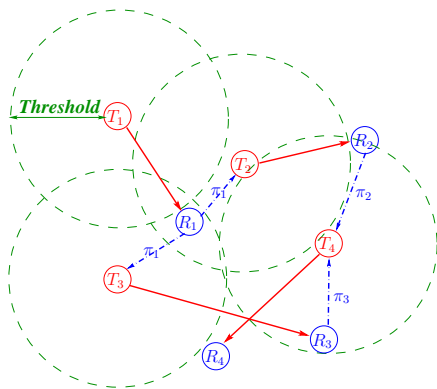
- ▶ this condition is always satisfied with log utilities.
- ▶ proof based on relating this algorithm to a “fictitious supermodular game.”

Comparison with Gradient-based Algorithm



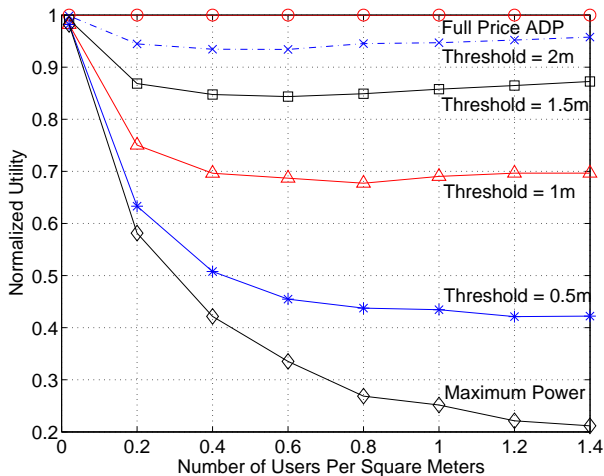
utility $\log(\gamma_i)$, 10 users

Limited Price Information Exchange



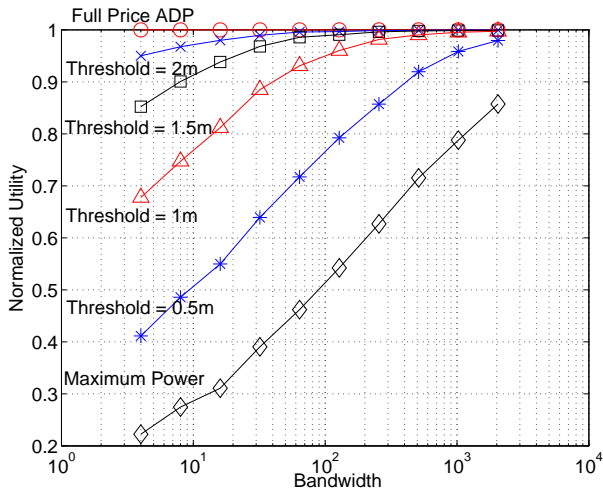
- Previously we assumed that each user can decode all prices.
- In practice, users (transmitters) may only decode prices within a **threshold** distance.

ADP Performance with Limited Exchange of Prices



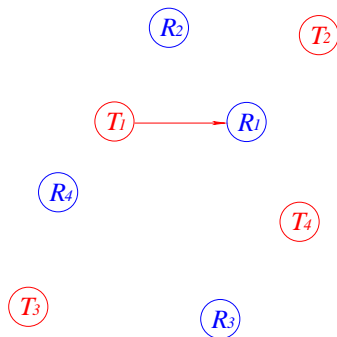
utility $\log(1 + \gamma_i)$, $10\text{m} \times 10\text{m}$ area

ADP Performance with Limited Exchange of Prices



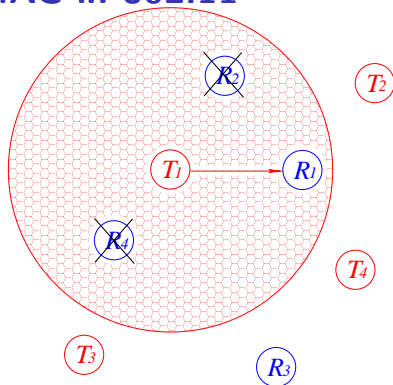
utility $\log(1 + \gamma_i)$, 1.4 users/ m^2

RTS/CTS MAC in 802.11



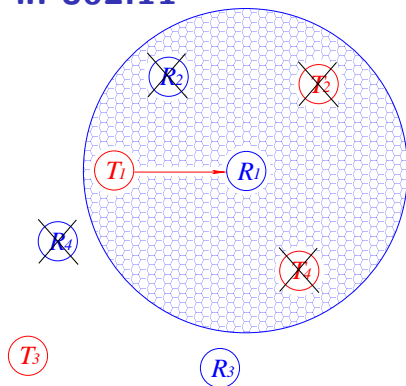
- Active users transmit at maximum power.
- Avoid interference with Request to Send (RTS) and Clear to Send (CTS) messages.
- Nodes are added sequentially, and are uniformly distributed over the region.

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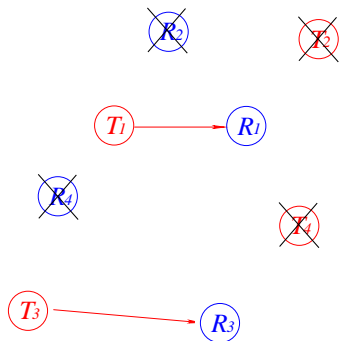
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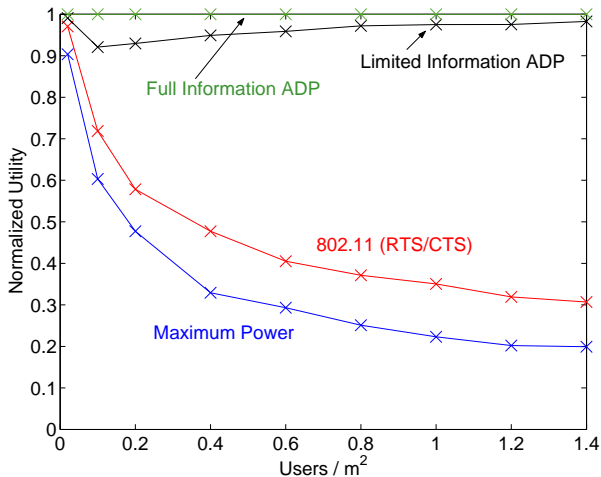
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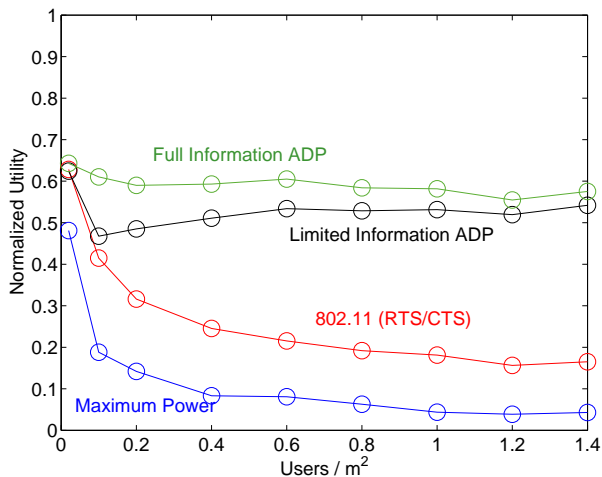
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Perfect Rate Control ADP vs. 802.11



utility $\log(1 + \gamma_i)$, 10m \times 10m area

Quantized Rate Control ADP vs. 802.11



quantized utility $\log(1 + \gamma_i)$ ($\{0, 5, 10, 15, 20\}$ bits/Hz), 10m \times 10m area

Conclusions

- Presented algorithm for **distributed** power control with limited information exchange in spread spectrum wireless (ad hoc) networks.
- Showed **optimality** and **convergence** for “sufficiently” concave utilities.
- Fast convergence and robust performance under practical conditions.
 - ▶ Performance degrades gracefully with decreasing threshold for decoding prices.
 - ▶ Information exchange provides substantial performance improvement relative to RTS/CTS MAC.
- Have also extended this to multi-channel model (Allerton paper).