Simple, Optimal, Fast, and Robust Wireless Random Medium Access Control

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Personal Research Background

- **Wireless Communications & Networking**
  - Cognitive Radio
  - Cooperative Communications
  - OFDM/CDMA Networks
  - Wireless Multimedia
  - Wireless MAC
- **Network Management & Economics**
  - Pricing & Revenue Management
  - Service Provider Competitions
  - Network Disruption Management
  - Robust Network Optimization

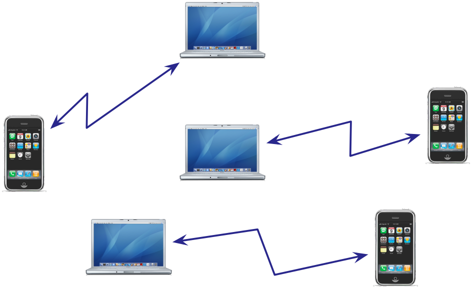
Key Methodologies

- Game Theory & Microeconomics
- Nonlinear Optimization
- Queueing & Stochastic Control

Wireless Random MAC

Ack: A. Mohsenian-Rad, M. Chiang, V. Wong

Wireless MAC Protocols



- Coordinate multiple wireless users accessing the same channel
  - Centralized: scheduling-based MAC (e.g., cellular network)
  - Distributed: contention-based random MAC (e.g., ad hoc network)

History of Wireless Random MAC

- Studied for 30 years
- Simplicity and practicality
- Many variations
    - Some achieved great success: Aloha, CSMA, ...
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  - Many are engineering ad hoc designs
- Two approaches:
  - **Reverse Engineering**: understand the math behind existing protocols [JSAC-07]
  - **Forward Engineering**: design better protocols [This Talk]

Forward Engineering

- How to design a better algorithm

Forward Engineering

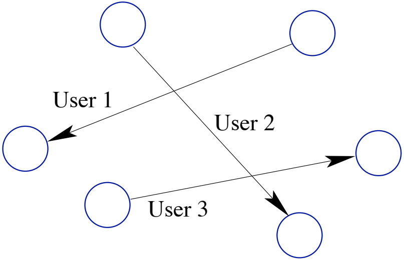
- How to design a better algorithm
- Simple, Optimal, Fast, and Robust

Forward Engineering

- How to design a better algorithm
- Simple, Optimal, Fast, and Robust
- Overcome performance bottlenecks of many previous algorithms

Our focus: Aloha

(A Simple) Network Model



- A set of $\mathcal{N} = \{1, \dots, N\}$ single-hop users
- Full interference topology (relaxed later)
- Each user i
  - Contend the channel with probability $p_i \in \mathcal{P}_i = [P_i^{\min}, P_i^{\max}]$
  - Maximum data rate γ_i
  - Long term average data rate

$$r_i(\mathbf{p}) = \gamma_i p_i \prod_{j \in \mathcal{N} \setminus \{i\}} (1 - p_j)$$

Network Utility Maximization

- Each user i has an increasing and concave utility function $u_i(r_i)$

System Objective: Network Utility Maximization (NUM)

$$\max_{\mathbf{p}\in\mathcal{P}} \sum_{i\in\mathcal{N}} u_i(r_i(\mathbf{p})),$$

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- We will focus on the α-fair utility function:

$$u_i(x) = \begin{cases} (1-\alpha)^{-1}\, x^{1-\alpha}, & \text{if } \alpha \in (0,1)\bigcup(1,\infty), \\ \log x, & \text{if } \alpha = 1. \end{cases}$$

  - $\alpha \rightarrow 0$: system throughput maximization
  - $\alpha = 1$: proportional fair allocation
  - $\alpha \rightarrow \infty$: max-min fairness

Previous Work

- Lee, Chiang, Canderbank 2007
- Wang, Kar 2006
- Chen, Low, Doyle 2005
- Gupta, Stolyar 2006

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- Several performance bottlenecks

Technical Challenges / Performance Bottlenecks

- Non-convexity: $\alpha \in (0,1)$ is an open problem even centrally
- No centralized controller $\Rightarrow$ need to be distributed and asynchronous
- Wireless lossy channels $\Rightarrow$ messages may get delayed and dropped
- Channels can be time varying $\Rightarrow$ demand fast convergence

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We will address these challenges

Key idea: Localize the Global Optimization Problem

- Intuition: each user optimizes the total network utility ⇒ solve NUM
- Challenge: what will be the information needed?

Local Optimization Problem

User *i*'s Local NUM Problem

$$\max_{p_i \in \mathcal{P}_i} \sum_{j \in \mathcal{N}} u_j(r_j(p_i, \mathbf{p}_{-i})),$$

- Objective: total network utility
- Variable: user *i*'s transmission probability p_i
- Parameter: other users' transmission probabilities

$$\mathbf{p}_{-i} = (p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_N)$$

Optimal Solution of Local Optimization

$$p_i^*(\mathbf{p}_{-i}) = f_i(\mathbf{p}_{-i}) = \left[1/\left(1+\sqrt[\alpha]{v_i(\mathbf{p}_{-i})}\,\right)\right]_{P_i^{\min}}^{P_i^{\max}},$$

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- $v_i(\mathbf{p}_{-i}) = {\gamma_i}^{\alpha-1} \sum_{j \in \mathcal{N} \setminus \{i\}} m_j$

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- $m_j = (1/\gamma_j)^{\alpha-1} (1/p_j - 1)^{\alpha-1}, \quad \forall j \in \mathcal{N}$
- If each j broadcasts the message $m_j \Rightarrow$ user i can calculate $p_i^*(\mathbf{p}_{-i})$.

Local Algorithm for User i

```
Initialize $p_i$ and $\mathbf{m} = (m_1, \cdots, m_N)$.
repeat
    Transmit with probability $p_i$.
    At a randomly chosen time, Update
        $p_i = \left[ 1 / \left( 1 + \sqrt[\alpha]{{\gamma_i}^{\alpha-1} \sum_{j \in \mathcal{N} \setminus \{i\}} m_j} \right) \right]_{P_i^{\min}}^{P_i^{\max}}.$
    At a randomly chosen time, update and broadcast
        $m_i = (1/\gamma_i)^{\alpha-1} (1/p_i - 1)^{\alpha-1}.$
until the user decides to leave the network.
```

Algorithm Properties

Theorem

Under proper technical conditions and for any α-fair utility function:

1. **Uniqueness**: the algorithm has a unique fixed point.
2. **Optimality**: it is also the unique global optimal solution of NUM problem.
3. **Convergence**: the algorithm globally and asynchronously converges.
4. **Robustness**: convergence is robust to any bounded message delay/loss.

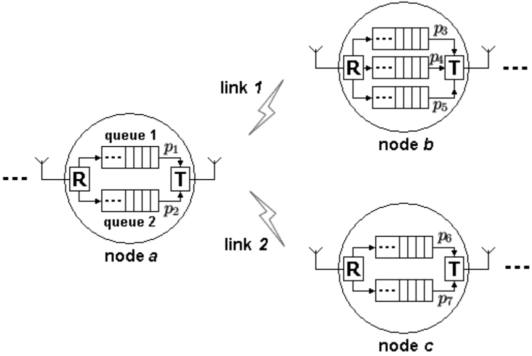
Key Maths

1. **Uniqueness**: contraction mapping or monotonic mapping

$$\mathbf{f}(\mathbf{p}) = (f_i(\mathbf{p}_i), \forall i)$$

2. **Optimality**: fixed point set of algorithm = KKT point set of NUM
3. **Convergence** & **Robustness**:
   - Synchronous convergence
   - Box condition
4. No convexity is required
   - The proposed algorithm works with enough contention level

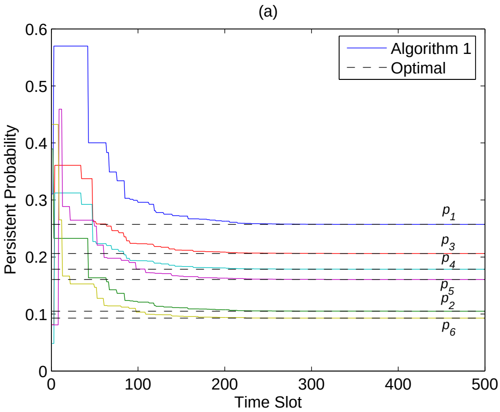
Extensions



- Can also be extended to general interference case
    - A node can have multiple outgoing links
    - A link may only interfere with a subset of other links
- All previous results go through.

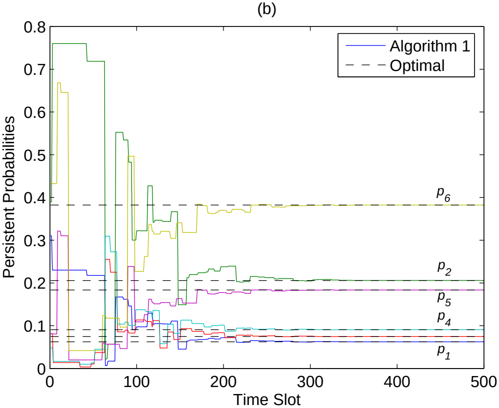
Convergence and Optimality

- 3 nodes and 6 links; $\alpha = 2$ (convex NUM)

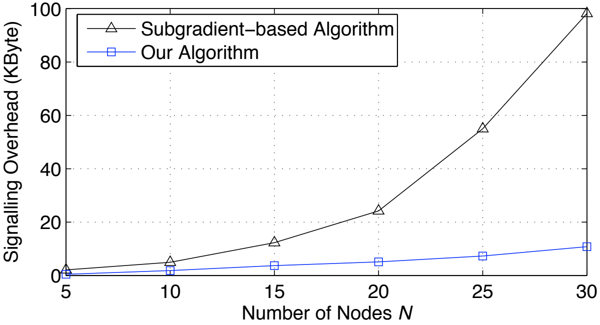


Convergence and Optimality

- 3 nodes and 6 links; $\alpha = 0.6$ (non-convex NUM)

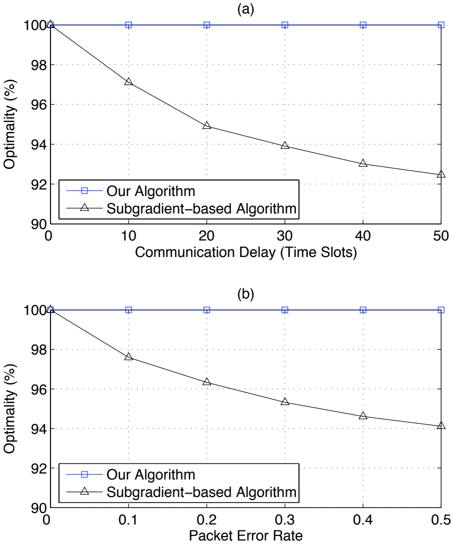


Signalling Overhead



Subgradient-based algorithm: J. Lee, M. Chiang, and R. Calderbank, "Utility-optimal random-access control," IEEE Trans. Wireless Comm., 2007.

Impact of Delay and Message Loss



Summary

- **Topic**: Forward engineering MAC as an optimization problem
- **Algorithm**: distributed asynchronous updates with limited message passing
- **Properties**: simple, optimal, fast, and robust
- **Extension**: the same algorithm works without any explicit message passing (with limited topology)

Related Journal Publications

- A. H. Mohsenian-Rad, J. Huang, M. Chiang and V.W.S. Wong, "Utility-Optimal Random Access: Reduced Complexity, Fast Convergence, and Robust Performance," *IEEE Transactions on Wireless Communications*, Feb. 2009
- A. H. Mohsenian-Rad, J. Huang, M. Chiang and V.W.S. Wong, "Utility-Optimal Random Access: Optimal Performance Without Frequent Explicit Message Passing," *IEEE Transactions on Wireless Communications*, March 2009

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