Distributed Resource Allocation in Communication Networks: From Competition to Cooperation

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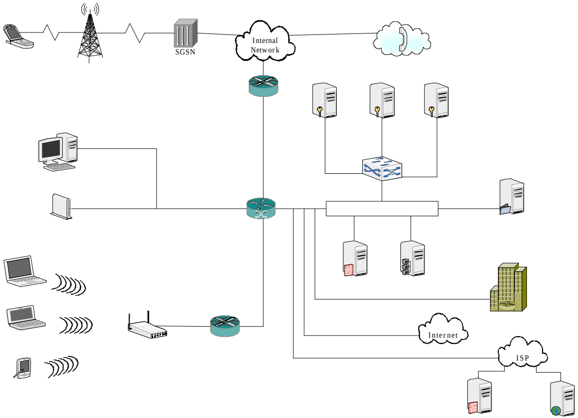
  - Hamilton Institute (Ireland): *Vijay Subramanian*
  - Marvell (Hong Kong): *Raphael Cendrillon*

Communication Networks Are Everywhere ...

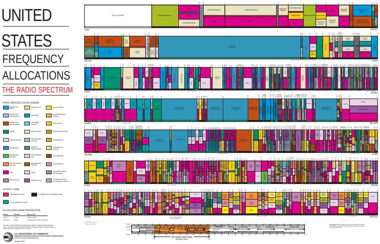


- Fast growth and successful deployment of
  - Internet: carrying data, voice, video, ...
  - Broadband access networks: DSL, cable, WiMAX, ...
  - Wireless networks: cellular, Wi-Fi, bluetooth, ...

... but Difficult to Design and Control

- Design Objectives:
  - Efficient utilization of the network resource
  - Fair opportunities for network access
- Research Challenges:
  - Physically distributed: self-interest, difficult to control centrally
  - Performance coupling: shared resource, mutual interactions

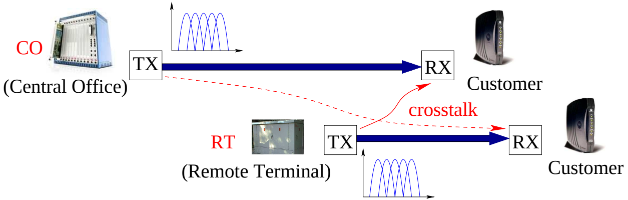
Example 1: Cognitive Radio Network (More Later)



Radio-domain-aware wireless network

- Project: Smart Markets for Smart Radio (Northwestern, Motorola)
- Physically distributed: devices learn and adapt to the environment
- Performance coupling: mutual interferences

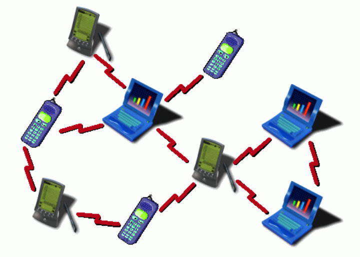
Example 2: DSL Network (More Later)



Copper-based last-mile broadband access network

- Project: FAST Copper (Princeton, Stanford, Fraser Research, AT&T)
- Physically distributed: phone lines terminate at different equipments
- Performance coupling: mutual interferences

Example 3: Mobile Ad Hoc Network (MANET)



Infrastructureless mobile wireless network

- Project: Control-based MANET (large DARPA Team, incl. Princeton)
- Physically distributed: nodes join and leave, move around
- Performance coupling: multihop relay, mutual interferences

We Need Good Resource Allocation Algorithms

- Need to design good resource allocation algorithms
  - Distributed
  - Low complexity
  - Local computation
  - Limited or no message passing
- Turn competition into cooperation

How to Achieve The Desired Solutions?

- **Mathematical Approaches**
  - Optimization theory
  - Game theory
  - Distributed computation and control
  - Microeconomics

- **Engineering Implications**
  - Realistic network deployments and tests
  - Engineering problem structure

Outline

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What is Cognitive Radio?

- Fixed Radio: transmit parameters (i.e., frequency, modulation) determined by hardware
- Adaptive Radio: parameters determined by software, easy to adapt to anticipated events
- **Cognitive Radio**: sense their environment and learn how to adapt

Why Cognitive Radio?

- Improve spectrum utilization
  - Licensed bands: IEEE 802.22 (Television band)
  - Unlicensed bands: IEEE 802.19
- Improve reliability
  - Emergency networks
  - Military networks
- Support interoperability
- Reduce costs of wireless communications

Spectrum Sharing for Licensed Band

- Primary owner: the exclusive licensee of the spectrum
- Secondary users: the cognitive radio devices
- Primary owner gets extra revenue by allowing secondary users to transmit in an unharmful way

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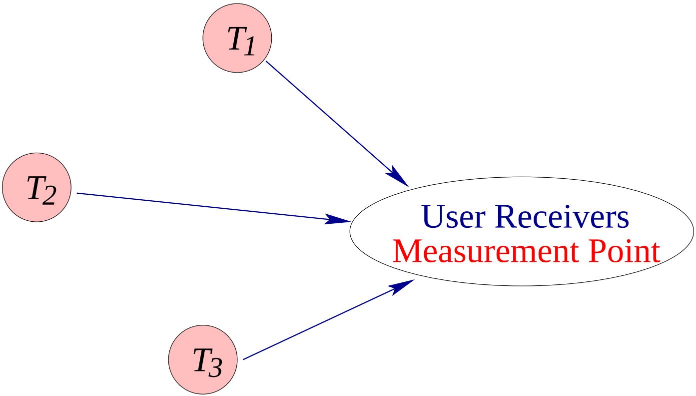
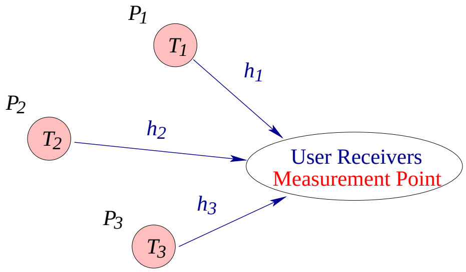
- **Interference temperature constraint**:
  - Maximum allowed interference measured at a measurement point
  - Equivalent to a total received power constraint

Network Model



More general model (with no new math challenges) will be discussed later

Mathematical Model

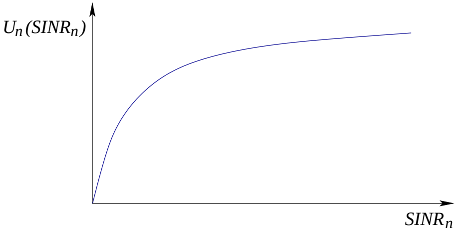


- Multi-user interference channel
- User n's
  - Transmission power: p_n
  - Channel gain: h_n
  - Signal-to-interference plus noise ratio (SINR):

$$\mathtt{SINR}_n(\mathbf{p}) = \frac{p_n h_n}{n_0 + \frac{1}{B}\left(\sum_{m \neq n} p_m h_m\right)}$$

- Interference temperature constraint: $\sum_n p_n h_n \leq P$

Utility Function



- Characterize QoS as function of SINR
- Increasing and strictly concave: elastic data application
- Private user information $\Rightarrow$ challenges to distributed solution

Network Objective I: Efficiency

- **Efficiency**: maximize the total network utility:

Efficiency Problem

$$\begin{aligned} \text{maximize} \quad & \sum_n U_n(\mathtt{SINR}_n(\mathbf{p})) \\ \text{subject to} \quad & \sum_n p_n h_n \le P \\ \text{variables} \quad & p_n \ge 0, \forall n \end{aligned}$$

- Example: $U_n(\mathtt{SINR}_n) = \theta_n \log(1 + \mathtt{SINR}_n)$
  - Maximizing total weighted rate

Network Objective II: Fairness

- **Fairness**: fair share of resource, independent of location

Fairness Problem

$$\begin{aligned} \text{maximize}\ & \mathtt{SINR}_1(\mathbf{p}) \\ \text{subject to}\ & U_n'(\mathtt{SINR}_n(\mathbf{p})) = U_m'(\mathtt{SINR}_m(\mathbf{p})), \forall m \neq n \\ & \sum_n p_n h_n \leq P \\ \text{variables}\ & p_n \geq 0, \forall n \end{aligned}$$

- Example: $U_n(\mathtt{SINR}_n) = \theta_n \log(\mathtt{SINR}_n)$
  - Weighted max-min fair

Technical Challenges

- Non-convexity:
  - SINR and utility may not be concave in power
- Physically distributed:
  - Local information: utility functions, channel gains
  - Selfish objectives
- Performance coupling:
  - Mutual interference
  - Shared received power at measurement point

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- Performance coupling:
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- **Our solution**: auction-based resource allocation algorithm
  - Distributed in nature
  - Capture interactions between users

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  - Allocation: how auctioneer allocates the good to the bidders
  - Payments: how the bidders pay the auctioneer

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- Types of auction
  - Indivisible auction
  - Divisible auction: suitable for communication resource allocation

Auction-based Comm. Resource Allocation

Network	Coupling	Bid	Efficiency	Rep. Paper
Wireline	simple	complex	Yes	Lazar-Semret'98
Wireline	simple	simple	Yes	Maheswaran-Basar'98
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- We design two auctions: efficient and fair allocation
- Proposed framework is general

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Initialization: manager announces

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Rules:

- Bids: user n submits $b_n \geq 0$ to the manager
- Allocation: the manager allows user n to generate interference at the measurement point with

$$p_n h_n = \frac{b_n}{\sum_n b_n + \beta} P$$

  - Weighted proportional allocation rule
  - Positive reserve bid makes sure that the values of bids count, not just the ratio

Two Different Payments

- **SINR auction**: user n pays $C_n(\pi) = \pi \times \mathtt{SINR}_n$
  - User-centric payment
  - Proportional to user's achieved QoS (SINR)
  - Leads to fair allocation

- **Power auction**: user n pays $C_n(\pi) = \pi \times p_n h_n$
  - Network-centric payment
  - Proportional to the allocated resource (power)
  - Leads to efficient allocation

Best Response and Nash Equilibrium

- Users participate in a non-cooperative game
  - User's **payoff** (benefit) = utility - payment
  - Both utility and payment depend on b_n and $b_{-n} \triangleq (b_m, \forall m \neq n)$

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- A user n wants to choose bid to maximize its own payoff
  - **Best response**: $\mathcal{B}_n(b_{-n})$

$$\mathcal{B}_n(b_{-n}) = \arg\max_{b_n} \left[U_n(\texttt{SINR}_n(b_n; b_{-n})) - C_n(\pi, b_n; b_{-n}) \right]$$

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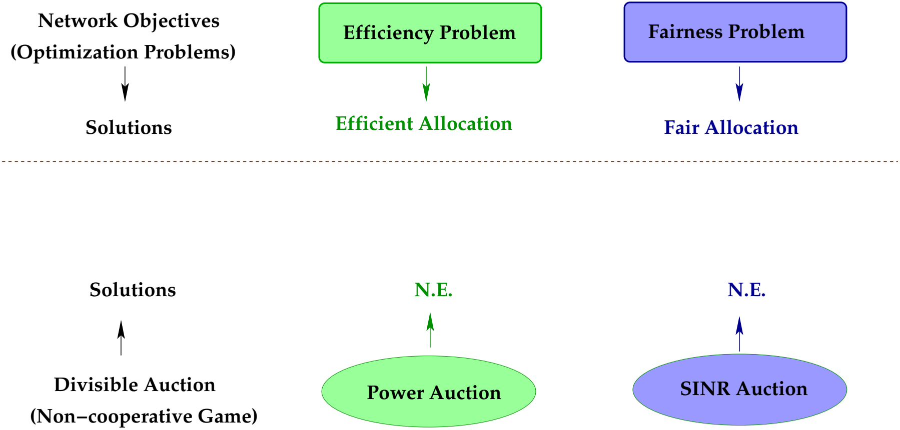
- Solution of the game: everyone is happy with the result
  - **Nash Equilibrium (N.E.)**: $\mathbf{b}^* = \{b_n^*, \forall n\}$
  - Fixed point solution of all users' best responses

$$b_n^* = \mathcal{B}\left(b_{-n}^*\right), \forall n$$

  - Stable outcome of the game



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Sufficiently Large Price Leads to Unique N.E.

Theorem (Sufficiently Large Price Leads to Unique N.E.)

In SINR Auction, there is a threshold price π_{th}, s.t.

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Proof: matrix analysis.

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3. Unique fixed point if spectral radius $\rho(\mathbf{K}(\pi)) < 1$

$$\mathbf{b}^* = \mathcal{B}(\mathbf{b}^*) \;\Rightarrow\; \mathbf{b}^* = \left(\sum_{i=0}^{\infty} \mathbf{K}^i\right) \mathbf{k}_0(\pi)\beta \;\; \textit{(N.E.)}$$

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4. Determine spectral radius through max-min operation

$$\rho(\mathbf{K}(\pi)) = \max_{\mathbf{x} \geq \mathbf{0}, \mathbf{x} \neq \mathbf{0}} \min_{m, x_m \neq 0} \frac{1}{x_m} \sum_{m=1}^{N} k_{mn}(\pi) x_n$$

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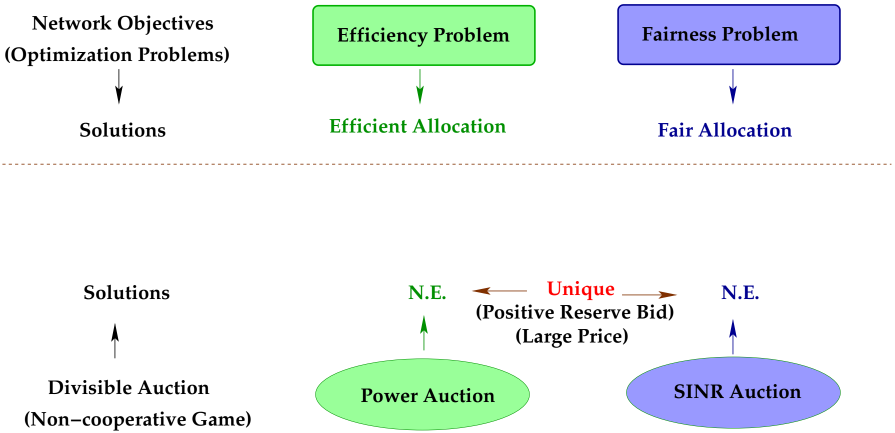
8. There exists $\pi_{th} \in [\underline{\pi}, \overline{\pi}]$, s.t. $\rho(\mathbf{K}(\pi)) < 1$ iff $\pi > \pi_{th}$.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SINR Auction: N.E. Achieves Fair Allocation

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Implication:

- Positive reserve bid β leads to resource waste
- This waste can be made very small by properly chosen price
- Argument can be made precise by defining an ϵ-system

Proof:

1. Satisfy fairness conditions (equalizing marginal utility)
2. Minimum $\texttt{SINR}_n$ can not be further improved

Power Auction: N.E. Achieves Efficient Allocation

- Large bandwidth ⇒ the efficiency problem becomes convex
  - Example: logarithmic utility $U_n(\mathtt{SINR}_n) = \log(\mathtt{SINR}_n)$

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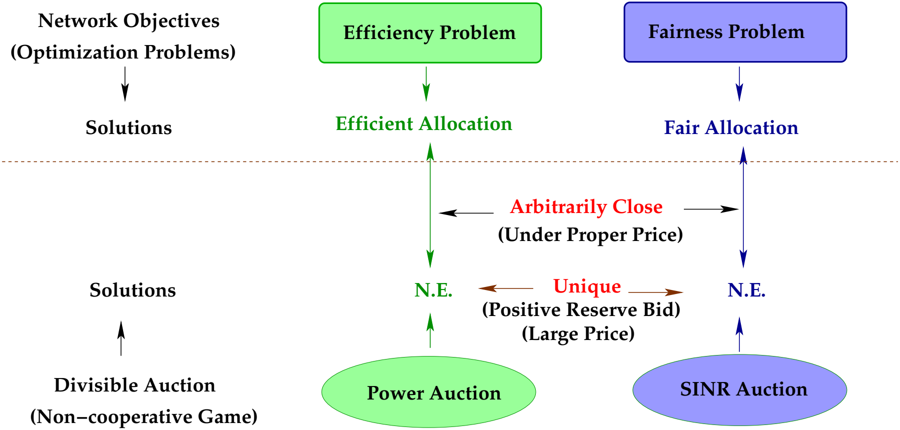
Proof:

1. Power allocation at the N.E. satisfies KKT conditions.
2. KKT conditions are necessary and sufficient for efficient allocation.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Theorem (BR Updates Locally Computable)

Best response update of user n can be written as

$$b_n^{(t)} = g_n^{(t-1)} b_n^{(t-1)}$$

where coefficient $g_n^{(t-1)}$ only depends on

- *Common information: P, n_0 and π.*
- *Local information: U_n, h_n and $\mathtt{SINR}_n^{(t-1)}$.*

Convergence of Best Response Updates

- SINR Auction

Theorem (Global Convergence of BR Updates)

Under a fixed price, best response updates globally and geometrically converge to the unique N.E.

  - Only limited explicit message passing:
    - bid (user to manager) and resource allocation (manager to user)
  - No need to know anything about other users

Convergence of Best Response Updates

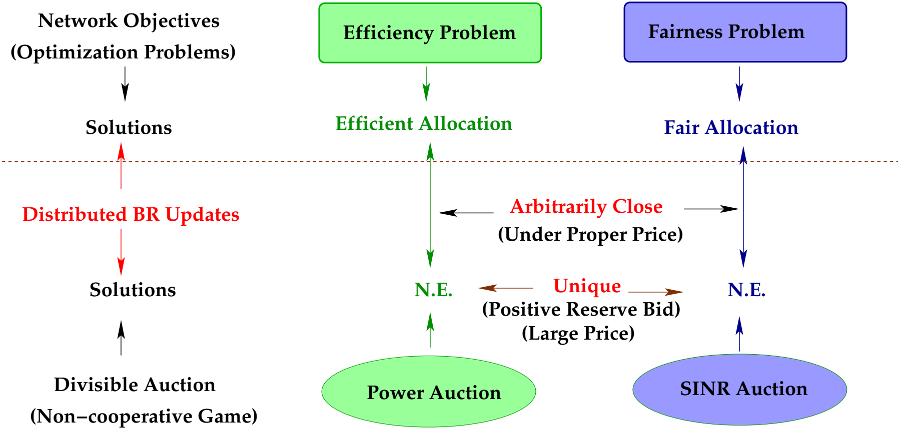
- SINR Auction

Theorem (Global Convergence of BR Updates)

Under a fixed price, best response updates globally and geometrically converge to the unique N.E.

  - Only limited explicit message passing:
    - bid (user to manager) and resource allocation (manager to user)
  - No need to know anything about other users

- Power auction
  - Similar arguments also apply for the power auction
  - Only works for specific utility functions
    - Examples: $\log(1 + \mathtt{SINR}_n)$ and $\log(\mathtt{SINR}_n)$


Network Objectives
(Optimization Problems)
Efficiency Problem
Fairness Problem
Solutions
Efficient Allocation
Fair Allocation
Distributed BR Updates
Arbitrarily Close
(Under Proper Price)
Solutions
N.E.
Unique
(Positive Reserve Bid)
(Large Price)
N.E.
Divisible Auction
(Non−cooperative Game)
Power Auction
SINR Auction

Extension to General Network Model

- All results related to SINR auction go through

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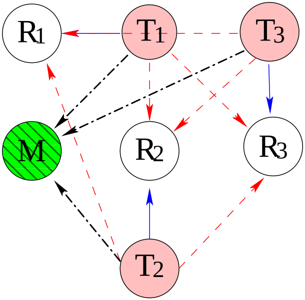
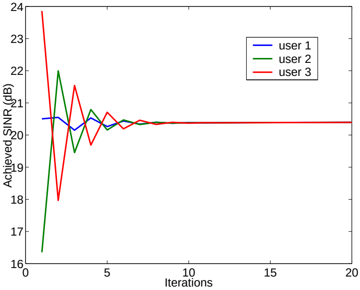


Network Topology



Convergence of SINR

- SINR Auction
- Same logarithmic utility function ⇒ same SINR allocation

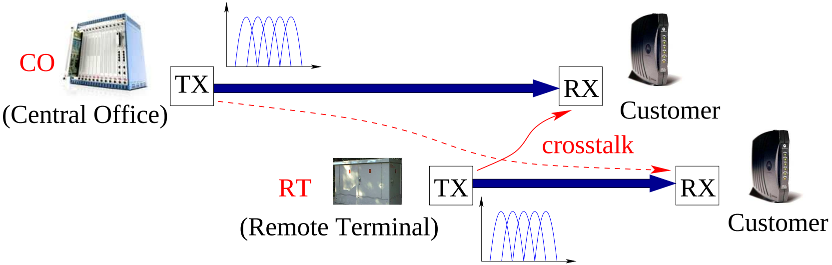
Summary

- **Topic**: spectrum sharing in licensed bands
- **Key idea**: simple divisible auction
- **Performance**
  - SINR auction: fair allocation
  - Power auction: efficient allocation
  - Large system: two auctions are both efficient
- **Algorithm**: best response updates: distributed, provable convergence
- **Practice**: a first step towards building a flexible cognitive-radio based spectrum sharing network
- **Main contribution**: a new modeling framework and solution methodology for distributed resource allocation in a coupled system

Outline

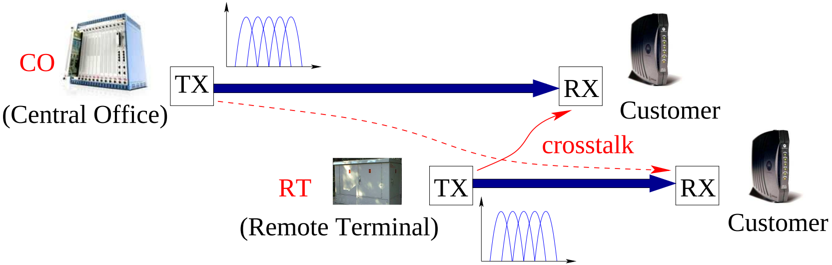
Digitial Subscriber Line (DSL) Networks

- Wireline communications networks based telephone copper lines
- Cost-effective broadband access network
- More than 160 million users world-wide



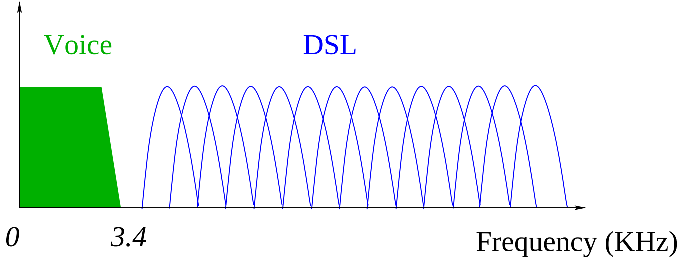
Digitial Subscriber Line (DSL) Networks

- Wireline communications networks based telephone copper lines
- Cost-effective broadband access network
- More than 160 million users world-wide
- Speed is the bottleneck

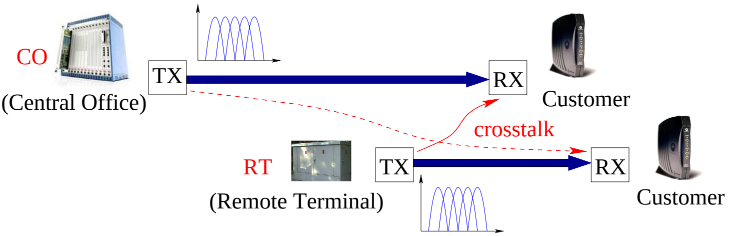


How DSL Works?

- Copper line can support signal transmissions over a large bandwidth
- Voice transmission: up to 3.4 KHz
- DSL transmissions: up to 30 MHz
  - Multi-carrier transmissions: Discrete Multitone Modulation



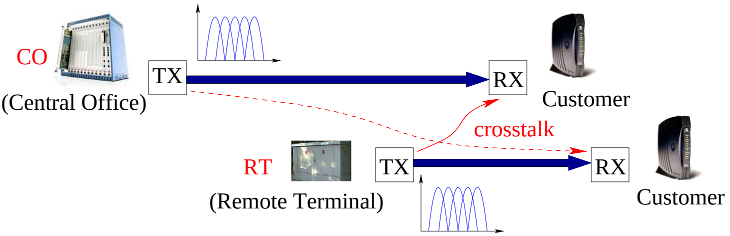
Network and Channel Model



Mathematical model: multi-user multi-carrier interference channel

- Each telephone line is a user (transmitter-receiver pair)
- Generate mutual crosstalks over multiple frequency tones

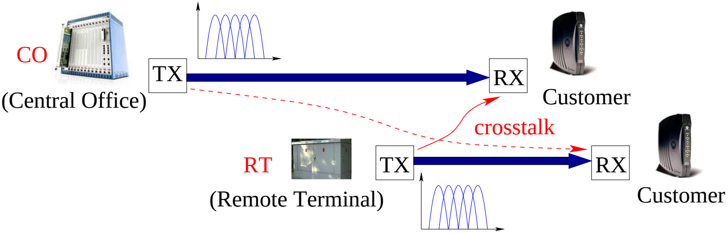
Network and Channel Model



Physical model: mixed CO/RT case

- Channel attenuates with distance
- Central Office (CO) connect customers who are reasonably close
- Remote Terminal (RT) connect customers who are farther away

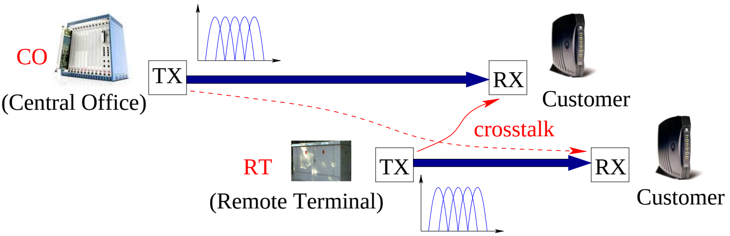
Network and Channel Model



Frequency-Dependent Channel

- Direct channel gain decreases with frequency
- Crosstalk channel gain increases with frequency

Network and Channel Model

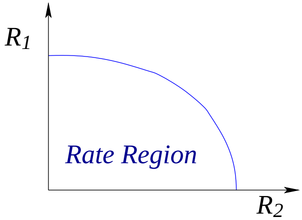


Frequency-Dependent Channel

- Direct channel gain decreases with frequency
- Crosstalk channel gain increases with frequency
- Lead to near-far problem
  - RT generates strong crosstalk to CO line, especially in high tones
  - CO generates little crosstalk to RT in all tones

Network Objective: Maximize Rate Region

Rate Region: set of all achievable rate vectors



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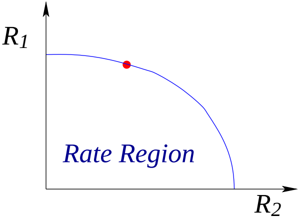

Problem A: (Find One Point On the Rate Region Boundary)

$$\underset{\{\mathbf{p}_n \in \mathcal{P}_n\}_n}{\text{maximize}} \sum_n w_n R_n$$

- User n's achievable rate $R_n = \sum_k \log\left(1 + \mathtt{SINR}_n^k\right)$.
  - No multi-user joint decoding
- User n chooses a power vector $\mathbf{p}_n \in \mathcal{P}_n = \left\{\sum_k p_n^k \le P_n^{\max}, p_n^k \ge 0\right\}$.

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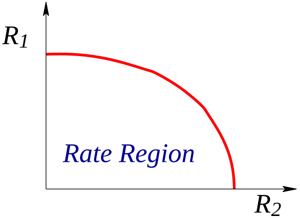

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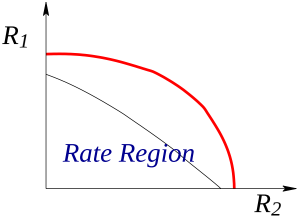

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- Changing different weights trace the entire rate region boundary
- A suboptimal algorithm leads to a reduced rate region

Properties of Problem A

- **Technical difficulties**
  - Non-convexity: total weighted rate not concave in power.
  - Physically distributed: local channel information
  - Performance coupling: across users (interferences) and tones (power constraint)

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  - Static channels
  - Multi-carrier transmissions
  - Typical network topology
  - Unique channel frequency responses with good empirical models
  - Cannot decode explicit message passing

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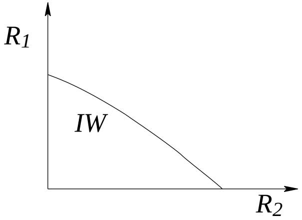
- **Our Solution**: ASB algorithm

Dynamic Spectrum Management (DSM)

- State-of-art DSM algorithms:
  - IW: Iterative Water-filling [Yu, Ginis, Cioffi'02]



Algorithm	Operation	Complexity	Performance
IW	Autonomous	$O(KN)$	Suboptimal

Dynamic Spectrum Management (DSM)

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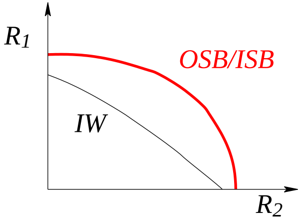

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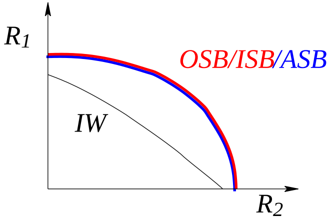
  - ASB: Autonomous Spectrum Balancing [Huang et al.'06]



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IW	Autonomous	$O(KN)$	Suboptimal
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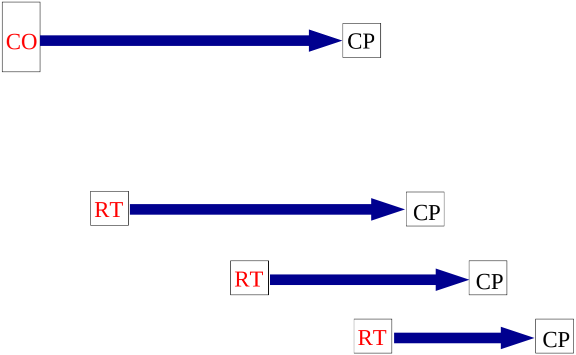
ASB	Autonomous	$O(KN)$	Near Optimal

Key Idea: Reference Line

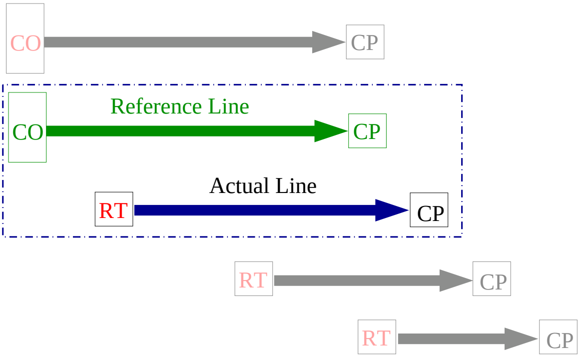
- **Reference line**: static pricing for static channel
  - A virtual line representative of the typical victim in the network
  - Good choice: the longest CO line
  - Parameters (power, noise, crosstalk) are publicly known

- Each user will choose its transmit power to protect the reference line

Reference Line

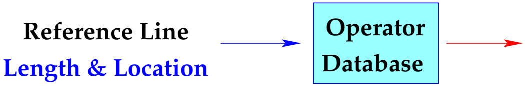


Reference Line



Reference Line's Rate

- User n's obtains the reference line parameters locally



Reference Power: $p^{k,ref}$

Reference Noise: $\sigma^{k,ref}$

Reference Crosstalk: $\alpha_n^{k,ref}$

Reference Line's Rate

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Reference Line
Length & Location → **Operator Database** →
Reference Power: $p^{k,ref}$
Reference Noise: $\sigma^{k,ref}$
Reference Crosstalk: $\alpha_n^{k,ref}$

- The reference line rate

$$R_n^{\text{ref}} = \sum_k \log\left(1 + \frac{p^{k,\text{ref}}}{\alpha_n^{k,\text{ref}} p_n^k + \sigma^{k,\text{ref}}}\right)$$

  - Interference only depends on user n's transmit power p_n^k
  - Locally computable without explicit message passing

Payoff and Best Response

- User n's **payoff**

$$S_n\left(\mathbf{p}_n;\mathbf{p}_{-n}\right) \triangleq R_n^{\text{ref}}\left(\mathbf{p}_n\right) + w_n R_n\left(\mathbf{p}_n;\mathbf{p}_{-n}\right)$$

Payoff and Best Response

- User n's **payoff**

$$S_n\left(\mathbf{p}_n;\mathbf{p}_{-n}\right)\triangleq R_n^{\text{ref}}\left(\mathbf{p}_n\right)+w_nR_n\left(\mathbf{p}_n;\mathbf{p}_{-n}\right)$$

- **Best response**

$$\mathcal{B}\left(\mathbf{p}_{-n}\right)\triangleq\arg\max_{\mathbf{p}_n\in\mathcal{P}_n}S_n\left(\mathbf{p}_n;\mathbf{p}_{-n}\right)$$

  - Requires solving a nonconvex optimization problem
  - Duality gap is zero (under large number of tones)
  - Satisfied in real DSL networks (ADSL: 256 tones, VDSL: 4096 tones)
  - Can be solved using dual decomposition

Frequency Selective Water-filling

- Under high SNR approximation of the reference line

$$\mathcal{B}_n^k(\mathbf{p}_{-n}) = \left(\frac{w_n}{\lambda_n + \alpha_n^{k,\text{ref}}/\sigma^{k,\text{ref}} \cdot \mathbf{1}_{\{p^{k,\text{ref}}>0\}}} - \sum_{m \neq n} h_{n,m}^k / h_{n,n}^k p_m^k - \sigma_n^k \right)^+$$

  - Reference line is not active in high frequency tones
- Special case: traditional water-filling (ignore $\alpha_n^{k,\text{ref}}/\sigma^{k,\text{ref}}$)

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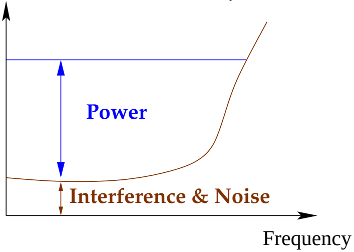

Traditional Water–Filling

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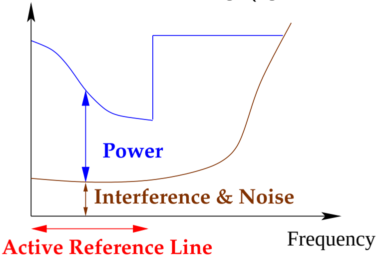


Frequency–Selective Water–Filling

Convergence of ASB Algorithm

- **ASB** Algorithm: users update their individual power allocation according to best responses either sequentially or in parallel

Theorem

ASB algorithm globally and geometrically converges to the unique N.E. if the crosstalk channel is small, i.e.,

$$\max_{n,m,k} \frac{h^k_{n,m}}{h^k_{n,n}} < \frac{1}{N-1}.$$

- Independent of the reference line parameters.
- Recover the convergence of iterative water-filling as a special case.

Proof: contraction mapping

Proof Outline

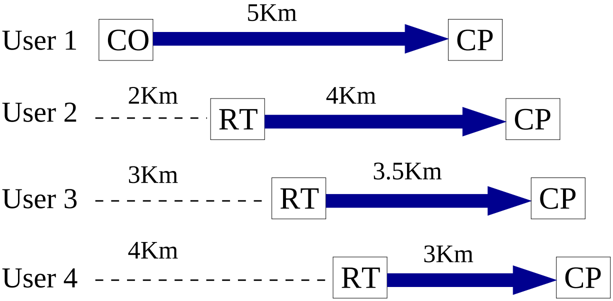
1. **Key Lemma**: min-max of an increasing function and an decreasing function is achieved at the intersection.
2. Construct two such functions based on the ASB algorithm.
3. Show the maximum difference between the PSD during adjacent iterations is decreasing.

$$\max_n \max \left\{ \sum_k \left[p_n^{k,t+1} - p_n^{t,t} \right]^+ , \sum_k \left[p_n^{k,t+1} - p_n^{k,t} \right]^- \right\}$$
$$\leq \max_n \max \left\{ \sum_k \left[p_n^{k,t} - p_n^{k,t-1} \right]^+ , \sum_k \left[p_n^{k,t} - p_n^{k,t-1} \right]^- \right\}$$

- Sequential updates: bound the maximum eigenvalue of the mapping matrix.
- Parallel updates: more realistic with cleaner proof.

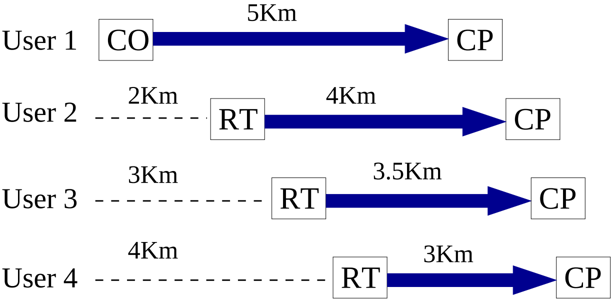
ASB Performance

- 4 ADSL lines.
- Mixed CO/RT deployment.
- Practical channel and background noise models.

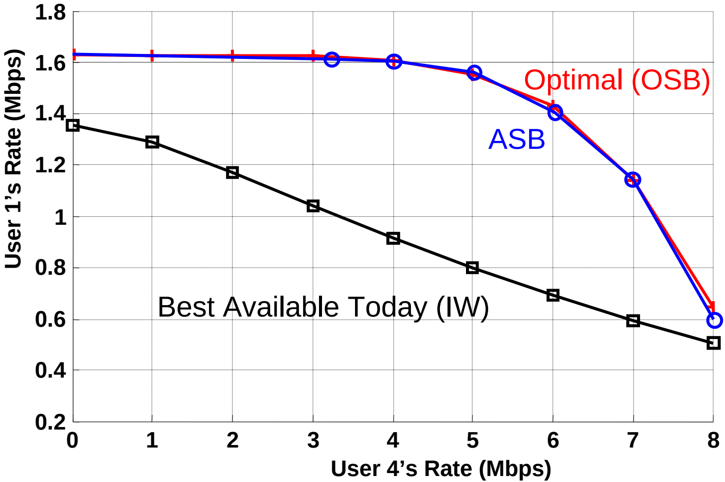


ASB Performance

- 4 ADSL lines.
- Mixed CO/RT deployment.
- Practical channel and background noise models.
- Both users 2 and 3 acheive fixed rates 2Mbps.
- Examine the rate region in terms of users 1 and 4's rates.



Achievable Rate Regions of Different Algorithms



Summary

- **Topic**: spectrum management in DSL multiuser interference channels
- **Key idea**: static pricing using reference line
- **Algorithm**: ASB: autonomous, low complexity, and robust
- **Performance**: close to optimal, provable convergence
- **Practice**: achieve significantly larger rate region compared with the state-of-the-art distributed algorithm

Summary: Key Technical Challenges

Performance Bottleneck: Non-convexity

- Mutual interference
- Integer constraints of channel utilization in OFDMA systems
- Discrete choices of multimedia coding and content selection

Complexity Bottleneck:

- Physically Distributed
  - Local utility information
  - Local channel/buffer information
  - Local multimedia content information
  - Self interests
- Coupled Performance
  - Mutual interference
  - Shared total received power
  - Shared total downlink transmission power
  - Shared relay resources or source budgets
  - Correlation between traffic arrival and departure

More Information

http://www.princeton.edu/~jianweih