

Managing Supply Uncertainty under Chain-to-Chain Competition

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We study the competition of two supply chains which are subject to supply uncertainty. Each supply chain consists of a supplier and a retailer. The retailers engage in Cournot competition by determining order quantities from their exclusive suppliers. The suppliers may not be able to fulfill the retailer orders at all time due to various causes. We examine the decisions of the suppliers and retailers at three different levels. At the operational level, we show that a retailer should order more (less) if its competing retailer has a less (more) reliable supply. Compared to chain competition without supply uncertainty, retailer's order sizes can be larger or smaller. At the design level, we consider two types of contracts and characterize their optimal contract terms under supply uncertainty. At the strategic level, we show that supply chain coordination is a dominant strategy, and customers are always better off. Nevertheless, supply chain coordination may or may not result in positive gains for the supply chain itself. If supply risk is low, coordination actually could decrease supply chain profit, which results in a prisoner's dilemma; If supply risk is high, coordination always increases supply chain profit.

Key Words: Supply uncertainty, supply chain contracting, Cournot competition.

1. Introduction

With the advances of new information technology and the increasing intensity of global competition, firms are working more closely with each other than ever to optimize their supply chains. As a result, the traditional model of firm-to-firm competition is giving way to a new paradigm of *chain-to-chain* competition (Barnes (2006), Ha and Tong (2008)). Moreover, supply uncertainty is becoming a major concern in the global supply chain management. The industrial survey conducted by Protiviti and APICS (American Production and Inventory Control Society) showed that 66% of respondents

considered supply interruption as one of the most significant concerns among all the supply chain related risks (O’Keeffe (2006)).

This article aims at offering a systematic examination on how to design and operate supply chains to effectively deal with supply uncertainty and chain-to-chain competition. In particular, we consider the following questions: how would supply uncertainty and competition affect firms’ ordering decisions? How to design contracts to effectively coordinate supply chains under risk and competition concerns? Is chain coordination desirable to all parties?

We explore these questions by studying the interaction of two competing supply chains. Each chain consists of one retailer and its exclusive supplier. Both chains are subject to supply uncertainty, i.e., the supplier may not be able to fulfill the order from the retailer due to external (e.g., snowstorm, custom delay) or internal causes (e.g., labor strike, machine breakdown). We consider two kinds of contracts that are popular in practice: the whole-sale price contract and revenue-sharing contract. Assuming linear penalty for supply disruption, we show that in general the whole-sale price contract can not coordinate a supply chain while the revenue-sharing contract may. Based on that, we consider three scenarios: i) the *coordinated competition game* between two coordinated chains; ii) the *hybrid competition game* between one uncoordinated chain and one coordinated chain; and iii) the *uncoordinated competition game* between two uncoordinated chains. We derive the equilibrium decisions of each game in terms of contract parameter choices and retailer order decisions. By comparing the three equilibria, we explore the impact of supply uncertainty and chain-to-chain competition on contract choices, supply chain profits, and customer benefits.

The key results of this paper are summarized as follows:

- *Operational level:* We show that a retailer should order more (less) if its competing retailer’s supply becomes less (more) reliable. Compared to the traditional supply chain competition setting without supply uncertainty, retailer’s order sizes can be larger or smaller.
- *Design level:* At the design level, we characterize the optimal contract terms for the whole-sale price contract and revenue sharing contracts with linear penalty for supply disruption. We show that in general the whole-sale price contract with linear penalty can not coordinate the supply chain, while a revenue sharing contract may.

- *Strategic level:* We show that supply chain coordination is always a dominant strategy under supply uncertainty and chain-to-chain competition. Moreover, customers are always better off in terms of expected market supply and price comparing with cases without coordination. Nevertheless, supply chain coordination may or may not result in positive gains for the supply chain itself. In particular, we show that in symmetric chains if supply risk is low, coordination actually could decrease supply chain profit, which results in a prisoner’s dilemma; If supply risk is high, coordination always increases supply chain profit.

The remainder of this paper is organized as follows. Section 2 reviews the related literature. Section 3 describes the model and introduces the notations. Section 4 studies the operational and design level decisions under three different game settings: the coordinated game, the hybrid game, and the uncoordinated game. Section 5 compares the three equilibria in Section 4 and derives strategic-level insights on contract choice, profit comparison and customer benefits. Finally we conclude in Section 6.

2. Literature

Our study is closely related to three areas: supply uncertainty, supply chain contracting, and supply chain competition.

The existing work on supply uncertainty can be divided into three categories: (i) the random-yield model, which models the uncertainty by assuming that the supply level is a random function of the input level (e.g., Yano and Lee (1995), Gerchak and Parlar (1990), Parlar and Wang (1993), Swaminathan and Shanthikumar (1999), Kazaz (2004), Babich et al. (2007), Federgruen and Yang (2008), Wang et al. (2008), Kazaz (2008), Deo and Corbett (2008)), (ii) the stochastic lead-time model, which models the lead-time as a random variable (see a comprehensive review in Zipkin (2000)), and (iii) the supply disruption model, which typically models the uncertainty of a supplier as one of two states: “up” or “down” (e.g., Arreola-Risa and DeCroix (1998), Gupta (1996), Meyer et al. (1979), Parlar and Berkin (1991), Song and Zipkin (1996), Tomlin (2006), Snyder and Shen (2006b), Snyder and Shen (2006a), Yang et al. (2009)). In particular, the orders are fulfilled on time

and in full when the supplier is “up”, and no order can be fulfilled when the supplier is “down”. Our proposed research builds upon the supplier disruption model.

Conflicts of interest is ubiquitous in a supply chain and contract is widely used to resolve these conflicts. The key task of supply contract design is to align the objectives of various firms with the supply chain’s overall objective and reduce the inefficiency of supply chains. Cachon (2003) provides an excellent review of if this field. There are several papers addressing the contracting and competition simultaneously, e.g., Parlar (1988), Cachon (2001), Corbett and Karmarkar (2001), Carr and Karmarkar (2005), Boyaci and Gallego (2004), and Ha and Tong (2008). In particular, Carr and Karmarkar (2005) examined the quantity competition under different supply chain structures with deterministic demand. Boyaci and Gallego (2004) studied two competing supply chains where the manufacturers and the retailers can take actions to affect service quality, and examined the value of coordination under full information. They showed that coordination is the dominant strategy but reduces the supply chain profit, as in the prisoner’s dilemma. Ha and Tong (2008) investigated two competing supply chains where only the retailers take actions that directly affect market competition, and studied the value of information sharing and the corresponding contracting choices. However, none of these papers considered supply disruption risks in a competition environment.

Only few papers have studied the impact of supply uncertainty in a competitive environment. Motivated by the case of the U.S. influenza vaccine market, Deo and Corbett (2008) examined the interaction between supply uncertainty and firm’s strategic decision on market entry. They found that supply uncertainty can contribute to a high degree of concentration in an industry and a reduction in the industry output and the expected consumer surplus in equilibrium. Babich et al. (2007) studied a supply chain where one retailer deals with competing risky suppliers who may default during their production lead times. They show that low supplier default correlations dampen competition among the suppliers, increasing the equilibrium wholesale prices. Therefore the retailer prefers suppliers with highly correlated default events, despite the loss of diversification benefits. In contrast, the suppliers and the channel prefer defaults that are negatively correlated. Hopp et al. (2008) investigated the impact of regional supply disruption on competing supply chains

where firms' strategies consist of two stages: (i) preparation, which involves investment prior to a disruption in measures that facilitate quick detection of a problem, and (ii) response, which involves post-disruption purchase of backup capacity for a component whose availability has been compromised. Using expected loss of profit due to lack of preparedness as a measure of risk, they found that the products that pose the greatest risk are those with valuable market share, low customer loyalty, and relatively limited backup capacity. Furthermore, they show that a dominant firm in the market should focus primarily on protecting its market share, while a weaker firm should focus on being ready to take advantage of a supply disruption to gain market share.

Our paper differs from the above-mentioned papers in that we focus on the ordering decisions, contract design and coordination choices under supply uncertainty and chain-to-chain competition. We contribute to the literature by offering a systematic examination on how to design and operate supply chains to effectively deal with supply risks and competition.

3. The Model

To investigate the impact of supply chain contracts on disruption management and competition, we consider the network model as in Fig. 1.

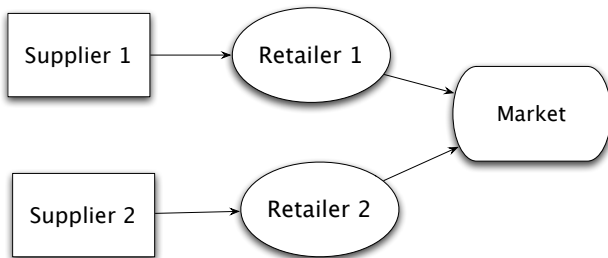


Figure 1: Model of Supply Chain Competition and Disruption

1. There are two competing supply chains that offer the same product in the market. Each chain consists of one retailer and one supplier. All parties are risk neutral. The two supply chains, suppliers, and retailers are indexed by i and j where $i, j \in \{1, 2\}$, $i \neq j$. The retailers engage in quantity competition.

2. A supplier provides the product to its retailer exclusively, and it is subject to disruption. We denote the reliability level of supplier i as α_i . With probabilities α_i , supplier i is “up” and fully fulfills retailer i ’s order; with probabilities $1 - \alpha_i$, supplier i is “down” and retailer i receives no inventory. We say the “reliability is high” or “disruption risk is low” if α_i is high. We consider the general case where two suppliers can have different reliability levels.
3. The product has a short life cycle and thus two supply chains only interact once. The market price of the product is determined by a function $p(x) = a - x$, where x is the total amount of inventory available in the market (i.e., the sum of the actual received deliveries by two retailers) and positive constant a characterizes the maximum market demand. This pricing function is widely used in quantity competition papers (e.g., in Ha & Tong (2008)). The function $p(x) = a - x$ reflects the fact that a larger supply leads to a lower market price.¹
4. We assume that the production costs are the same for both supply chains and are paid upon successful delivery. Without loss of generality, we let the unit production cost be scaled to zero.² As the unit production cost is scaled to be zero, a positive (negative) wholesale price charged by the supplier to the retailer means that the wholesale price is higher (lower) than the production cost.
5. We assume that the retailers have no option for backup (contingent) sourcing, supply uncertainty are independent of each other, and all parties have full information of the whole network.

Note that when $\alpha_i = \alpha_j = 0$, no retailer will order since suppliers can never deliver. When $\alpha_i = 0$ and $\alpha_j > 0$, i.e., one supplier always fails while the other not, the market degenerates to a monopoly case where only retailer j will order and might deliver to the market. In the monopoly case, the optimal order quantity Q_j^* is independent of α_j . Hence, for the rest of the paper we assume that $0 < \alpha_i \leq 1$ for $i = 1, 2$.

¹Note that when $p(x) = a - bx$, we can redefine $x' = bx$ and the analysis in this paper goes through without modification. We assume that a is a constant for the presentation clarity. However, little modification of the our analysis is needed when a is a random variable, which corresponds to the case where demand is subject to a random shock.

²When the production cost is positive (e.g., the unit cost $c > 0$), we can write $a' = a - c$ and the analysis follows, since $(a - x)x - cx = ((a - c) - x)x$.

The sequence of events is as follows ($i \in \{1, 2\}$), and both chains act simultaneously in each stage.

1. *Contract decision*: supplier i offers a contract to retailer i .
2. *Order decision*: retailer i decides order quantity Q_i from supplier i .
3. *Uncertain supply*: supplier i delivers the order Q_i to retailer i with probability α_i and zero with probability $1 - \alpha_i$.
4. *Quantity competition*: the market is cleared based on the total realized products from both supply chains and each party receives the payoff.

4. Design and Operation Decisions

In this section we will discuss the optimal operation and design decisions under three settings. First, we analyze the equilibrium for the coordinated competition game between two coordinated chains. Second, we derive the equilibrium for the hybrid competition game between a coordinated chain and an uncoordinated chain. Finally, we examine the equilibrium for the uncoordinated competition game between two uncoordinated chains. Comparisons between three settings and insights into the strategic decisions (coordinate or not) will be provided in Section 5.

4.1 Coordinated Competition Game

We first consider the case when both supply chains are coordinated, i.e., retailer and suppliers are fully aligned to achieve the supply chain's optimal performance. We first consider each chain as a central planner (instead of supplier and retailer as two decision makers), and analyze their quantity competition under supply disruptions. We seek to derive the order quantity for each supply chain at the market equilibrium. We then show that a revenue sharing contract can be used to achieve the coordination within each supply chain and thus there is no loss of assuming that one chain as one central planner.

4.1.1 Equilibrium between two coordinated chains

In this case, the two chains play a game by choosing the order sizes Q_1 and Q_2 .

Definition 1 (The Coordinated Competition Game). *A Coordinated Competition Game* $(\mathcal{I}, \{Q_i\}_{i \in \mathcal{I}}, \{E(\pi_i)\}_{i \in \mathcal{I}})$ is defined as

- *Players:* a set of two players $\mathcal{I} = \{1, 2\}$ representing two supply chain central planners,
- *Strategy:* each player i chooses the production quantity $Q_i \geq 0$,
- *Payoff:* each player i receives an expected profit $E(\pi_i(Q_i, Q_j))$.

The goal of the central planner i is to maximize the expected profit for supply chain i , which can be expressed as follows:

$$\begin{aligned} E(\pi_i(Q_i, Q_j)) &= E[p(Q_i I_i + Q_j I_j) Q_i I_i] \\ &= \alpha_i \alpha_j (a - Q_i - Q_j) Q_i + \alpha_i (1 - \alpha_j) (a - Q_i) Q_i \\ &= a \alpha_i Q_i - \alpha_i Q_i^2 - \alpha_i \alpha_j Q_i Q_j. \end{aligned}$$

Here I_i is an index variable, which equals 1 with probability α_i and 0 with probability $1 - \alpha_i$. The expected value of I_i reflects on average how reliable supplier i is.

The following theorem characterizes the unique pure strategy equilibrium and the corresponding players' payoffs of the Coordinated Competition game.

Theorem 1. *The unique pure strategy Nash equilibrium of the Coordinated Competition game is (Q_i^*, Q_j^*) , where the optimal order quantity of supply chain i is*

$$Q_i^* = \frac{(2 - \alpha_j)a}{(4 - \alpha_i \alpha_j)}, \quad \forall i, j \in \{1, 2\}, i \neq j; \quad (4.1)$$

The expected profit of supply chain i is

$$E(\pi_i^*) = \frac{\alpha_i (2 - \alpha_j)^2 a^2}{(4 - \alpha_i \alpha_j)^2} = \alpha_i (Q_i^*)^2. \quad (4.2)$$

The proof is provided in the Appendix. Note that in the absence of any disruption risks, i.e., $\alpha_i = \alpha_j = 1$, the optimal order quantities equal to $Q^* = a/3$ and the expected profits equal to $a^2/9$. Not surprisingly, this is exactly the result of a standard Cournot duopoly game. Moreover, it is easy to see that for any positive α_i and α_j , $Q^* < a/2$ and $E(\pi^*) < a^2/4$, i.e., the optimal order quantities and expected profits with supply disruption risks are bounded by those for the standard monopoly game without supply disruption. Following we obtain the comparative statics.

Corollary 1. *At the Nash equilibrium of the Coordinated Competition game, the expected total market supply is increasing in α_i and α_j . As a result, the expected market price is decreasing in α_i and α_j .*

The proof for Corollary 1 is as follows: $E(\text{total market supply}) = a_i a_j (Q_i^* + Q_j^*) + a_i (1 - a_j) Q_i^* + a_j (1 - a_i) Q_j^* = \frac{2a}{4 - \alpha_i \alpha_j} (a_i + a_j - \alpha_i a_j)$. Take first order derivative with respect to a_i and a_j , we have $\partial(\frac{2a}{4 - \alpha_i \alpha_j} (a_i + a_j - \alpha_i a_j)) / \partial a_i = 2 \frac{a}{(\alpha_i \alpha_j - 4)^2} (\alpha_j - 2)^2 > 0$. Similar results can be obtained for α_j . Because $E(p) = a - E(\text{total market supply})$, the expected market price decreases as supply chains become more reliable (i.e., α_i and α_j become larger).

Corollary 1 shows that total expected market supply achieves the maximum and the expected price reaches the minimum when there is no disruption ($\alpha_i = \alpha_j = 1$). This means that disruption actually hurts the customer, since the customers need to pay for a higher market price. Then how about the impact on the supply chains' performances? Corollary 2 answers this question.

Corollary 2. *At the Nash equilibrium of the Coordinated Competition game, the optimal order quantity for supply chain i is decreasing in α_j and increasing in α_i . The expected profit for supply chain i is decreasing in α_j and increasing in α_i .*

Corollary 2 indicates that a supply chain's profit and order size increase if its supplier is more reliable but decreases if its competitor's supplier becomes more reliable. We perform detailed analysis for both symmetric and asymmetric cases:

- *Special case of symmetric game:* We first consider the symmetric case where $0 < \alpha_i = \alpha_j = \alpha < 1$. It can be verified that as the supply reliability α decreases, the optimal order quantities

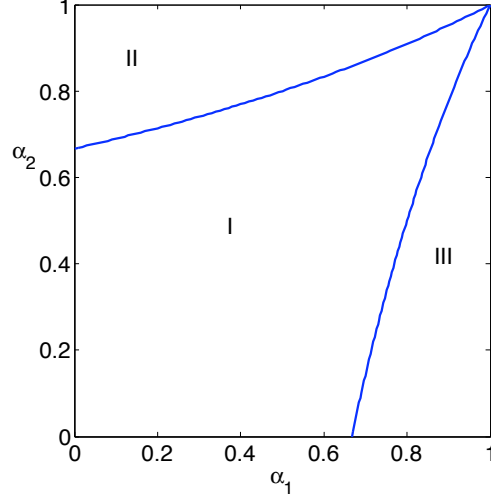


Figure 2: Supply Chain Order Sizes

increase (e.g., $Q^* = a(2 - \alpha)/(4 - \alpha^2) = a/(2 + \alpha)$ increases in α) and the expected supply chain profits decrease (e.g., $E(\pi^*) = \frac{\alpha(2-\alpha)^2 a^2}{(4-\alpha^2)^2} = \frac{\alpha a^2}{(2+\alpha)^2}$ decreases in α). Consequently, we can show that $a/3 \leq Q^* < a/2$ and $a^2/9 \leq E(\pi^*) < a^2/4$, e.g., the optimal order quantities and expected profits with supply disruption risks are bounded below by those for the standard Cournot duopoly game and above by those for the monopoly game without supply disruption.

- *General case of asymmetric game:* Now let us look at the case where the two supply chains have different values of α_i and α_j . Following are some observations about the optimal order sizes and expected profits.
 - The optimal order sizes under disruption can be larger or smaller than those in the classic Cournot competition without disruption (i.e., $a/3$). Fig. 2 illustrates this through three areas. In area (I), the reliability values of both chains are not drastically different, and both chains choose to aggressively order and their optimal order quantities are both larger than $a/3$. In area (II), chain 1 has a significantly higher reliability than chain 2, and it is optimal for chain 1 to order less than $a/3$ and for chain 2 to order more than $a/3$. Finally in area (III), the optimal order quantity for chain 1 is larger than $a/3$ while that for chain 2 is smaller than $a/3$.

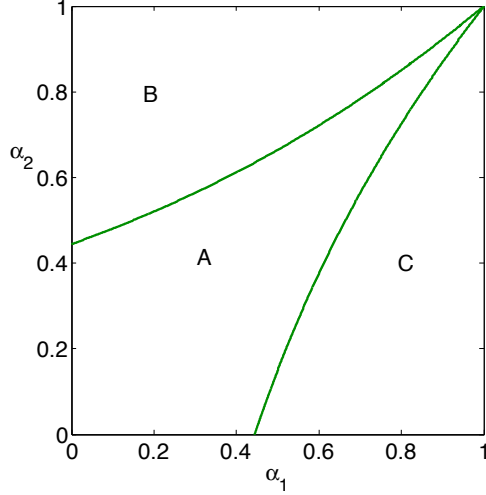


Figure 3: Supply Chain Profits

- The expected profits under disruption can be larger or smaller than those without disruption ($a^2/9$). Fig. 3 illustrates this through three regions (note that they are *different* from those in Fig. 2). In area (A), the expected profits for both chains are smaller than $a^2/9$. In area (B), the expected profit for chain 1 is smaller than $a^2/9$ while that for chain 2 is larger than $a^2/9$. Finally in area (C), the expected profit for chain 1 is larger than $a^2/9$ while that for chain 2 is smaller than $a^2/9$.

4.1.2 Coordinating Contract

We have characterized the equilibrium when both chains are coordinated. The question remains to be answered is what contract can actually coordinate the supply chain under disruption. In general, a contract coordinates the retailer's and the supplier's actions whenever each party's profit is an affine function of the supply chain's profit. In this paper, we will focus on two types of popular contracts: the revenue sharing contract and the wholesale-price contract. These contracts have been well studied in the literature (e.g., Cachon (2003)). The main difference in our paper is that we also consider penalty scheme for supply disruption. For simplicity of analysis, we assume that penalty is linear, i.e., the supplier compensates the retailer with $S \geq 0$ dollar for each unit that he fails to deliver. We leave the study of more complex contract forms for future research. In

the following we show that with appropriately chosen parameters the revenue sharing contract can coordinate supply chain under supply uncertainty. Later we will show that in general the whole-sale price contract with linear penalty cannot coordinate chain.

We consider a revenue sharing contract where the supplier's income consists of three parts: ω for each unit order that placed by the retailer, a percentage of retail revenue for each successfully delivered order, and a penalty for each unit order that the supplier fails to deliver. Let $\phi \in [0, 1]$ be the fraction of supply chain revenue the retailer keeps, so the supplier earns the remaining $(1 - \phi)$. The profit functions for the retailer, the supplier, and the central planner are

$$\begin{aligned} E(\pi_r) &= E[(\phi p - \omega)QI + SQ(1 - I)], \\ E(\pi_s) &= E\{[(1 - \phi)p + \omega]QI - SQ(1 - I)\}, \\ E(\pi) &= E(\pi_r) + E(\pi_s) = E(pQI). \end{aligned}$$

Here I is the index function that equals 1 with probability α . When $\omega - (1 - \alpha)S/\alpha = 0$, we have

$$E(\pi_r) = E[\phi p Q I] = \phi E(\pi),$$

i.e., the profit of the retailer is an affine function of the central planner. Hence such a contract coordinates the supply chain. Furthermore, ϕ decides how supply chain total profit is divided between the retailer and supplier.

4.2 Hybrid Competition Game

In this section, we study the case where one supply chain is coordinated while the other is not. We are interested in finding out how the equilibrium order quantities and the profits change compared to the coordinated game. In particular, will the coordinated chain have a competitive edge over the uncoordinated chain in the hybrid scenario?

We suppose that supply chain i uses an appropriately designed revenue sharing contract and is therefore coordinated. The profit of retailer i is

$$E(\pi_{i,r}(Q_i, Q_j)) = \phi \alpha_i Q_i (a - Q_i - \alpha_j Q_j), \quad (4.3)$$

where $0 < \phi < 1$ is a fixed contract parameter. For supply chain j , we assume that the supplier charges wholesale price ω per unit if he makes the delivery successfully, otherwise he pays a penalty

S per each unit of shortage. This is a linear contract that is easy to implement. The expected profit of retailer j is

$$\begin{aligned}
E(\pi_{j,r}(Q_i, Q_j, m)) &= E[(p - \omega)Q_j I_j + SQ_j(1 - I_j)] \\
&= E[p(Q_j)I_j]Q_j - E[\omega I_j - S(1 - I_j)]Q_j \\
&= \alpha_j Q_j(a - Q_j - \alpha_i Q_i) - mQ_j.
\end{aligned} \tag{4.4}$$

Here we define $m = \alpha_j \omega - (1 - \alpha_j)S$ as the supplier j 's *expected profit margin* for each unit ordered by the retailer, i.e., with probability α_j the supplier receives ω for each unit of successful delivery to the retailer and with probability $1 - \alpha_j$ the supplier pays S for each unit of shortage. The expected profit of supplier j is

$$E(\pi_{j,s}(Q_j, m)) = mQ_j. \tag{4.5}$$

If m is positive, it means that the supplier's expected profit margin is higher than his production cost (since we scale the production cost to be zero); otherwise the supplier subsidizes the retailer.

Will this whole sale plus penalty contract coordinates the supply chain? It is easy to show that it does coordinate if and only if $m = 0$, e.g., the supplier earns zero profit. This is clearly unacceptable to the supplier in practice, and thus such wholesale price contract does not coordinate the supply chain in the cases that we are interested. Thus we assume that chain j is the uncoordinated chain.

We use superscript "*h*" to denote the hybrid competition game when supply chain i is coordinated and supply chain j is not. Now let us define the game where supplier j can freely choose the contract m before the quantity competition happens. This is an extensive game with three players and two stages, defined as follows.

Definition 2 (Hybrid Competition Game). *A Hybrid Competition game is defined as*

- *Players: a set of three players: retailer i , retailer j , and supplier j ,*
- *There are two stages in the game*
 - *Stage 1: supplier j announces the contract term m to retailer j .*
 - *Stage 2: retailers i and j choose production quantities Q_i and Q_j simultaneously.*

- *Payoff*: the expected revenues for the three players are $E(\pi_{i,r}(Q_i, Q_j))$, $E(\pi_{j,r}(Q_i, Q_j, m))$, and $E(\pi_{j,s}(Q_j, m))$, respectively.

Notice that for a different contract term m offered by supplier j in the first stage, the game played by the central planner i and retailer j at the second stage is a different subgame. A subgame is any part (subset) of a game where it starts from a singleton information set (i.e., there is no ambiguity in terms of information among players) and includes all subsequent parts of the original game. For an extensive game, the most commonly used solution concept is subgame perfect Nash equilibrium, which is a refinement of the Nash equilibrium concept.

Definition 3 (Subgame Perfect Nash Equilibrium (SPNE)). *A Nash equilibrium is subgame perfect if the players' strategies constitute a Nash equilibrium in every subgame.*

The SPNE of the Hybrid Competition game can be characterized as follows.

Definition 4. *A pure strategy SPNE $(Q_i^h(m), Q_j^h(m), m^*)$ of the Hybrid Competition game satisfies*

1. *Retailers compete with each other in quantities to maximize their profits for any given contract m (not necessary optimal), i.e.,*

$$Q_i^h(m) \in \arg \max_{Q_i \geq 0} E(\pi_{i,r}(Q_i, Q_j^h(m))), \quad (4.6)$$

and

$$Q_j^h(m) \in \arg \max_{Q_j \geq 0} E(\pi_{j,r}(Q_i^h(m), Q_j, m)). \quad (4.7)$$

2. *Supplier j sets the contract term to maximize its expected profit given the choices of retailers, i.e.,*

$$m^* \in \arg \max_{m \geq 0} E(\pi_{j,s}(Q_j^h(m), m)). \quad (4.8)$$

The following lemma characterizes the equilibrium strategies for both retailers as a function of contract term m .

Lemma 1. *At a pure strategy SPNE of the Hybrid Competition game,*

i) If $0 \leq m < \frac{a\alpha_j(2-\alpha_i)}{2}$, then the equilibrium production quantity for supply chain i (the coordinated chain) is

$$Q_i^h(m) = Q_i^* + \frac{m}{4 - \alpha_i\alpha_j}, \quad (4.9)$$

and that of supply chain j (the uncoordinated chain) is given by

$$Q_j^h(m) = Q_j^* - \frac{2m}{\alpha_j(4 - \alpha_i\alpha_j)}. \quad (4.10)$$

Here Q_i^* and Q_j^* are the equilibrium production quantities in the Coordinated Competition game in Section 4.1.

ii) If $m \geq \frac{a\alpha_j(2-\alpha_i)}{2}$, then $Q_i^h(m) = \frac{a}{2}$ and $Q_j^h(m) = 0$.

From (4.9) and (4.10), we see that if m is positive, then coordinated retailer i orders more inventory in the hybrid game than in the coordinated game. When $m = 0$, the equilibrium for the hybrid scenario reduces to the case analyzed in Section 4.1, meaning that a sufficient condition to maximize the total profit of supply chain j is to have $m = 0$. Since this is not possible in practice, the un-coordinated chain j suffers from a lower level of order compared with a coordinated game.

Theorem 2. *At an SPNE of the Hybrid Competition game, supplier j chooses the contract terms ω^* and S^* such that*

$$m^* = \alpha_j\omega^* - (1 - \alpha_j)S^* = \frac{a\alpha_j(2 - \alpha_i)}{4} > 0. \quad (4.11)$$

Consequently, retailer i orders

$$Q_i^h(m^*) = \frac{a(8 - 2\alpha_j - \alpha_i\alpha_j)}{4(4 - \alpha_i\alpha_j)} > 0 \quad (4.12)$$

and retailer j orders

$$Q_j^h(m^*) = \frac{a(2 - \alpha_i)}{2(4 - \alpha_i\alpha_j)} > 0. \quad (4.13)$$

The expected profit for coordinated supply chain i is

$$\begin{aligned} E(\pi_i^h) &= \frac{\alpha_i a (8 - 2\alpha_j - \alpha_i\alpha_j)}{4(4 - \alpha_i\alpha_j)} \left(a - \frac{a(8 - 2\alpha_j - \alpha_i\alpha_j)}{4(4 - \alpha_i\alpha_j)} - \frac{\alpha_j a (2 - \alpha_i)}{2(4 - \alpha_i\alpha_j)} \right) \\ &= \alpha_i (Q_i^*)^2 \left(\frac{8 - 2\alpha_j - \alpha_i\alpha_j}{8 - 4\alpha_j} \right)^2, \end{aligned} \quad (4.14)$$

and that for uncoordinated supply chain j is

$$\begin{aligned} E(\pi_j^h) &= \frac{\alpha_j a(2 - \alpha_i)}{2(4 - \alpha_i \alpha_j)} \left(a - \frac{a(2 - \alpha_i)}{2(4 - \alpha_i \alpha_j)} - \frac{\alpha_i a(8 - 2\alpha_j - \alpha_i \alpha_j)}{4(4 - \alpha_i \alpha_j)} \right) \\ &= \alpha_j (Q_j^*)^2 \left(\frac{6 - \alpha_i \alpha_j}{8} \right), \end{aligned} \quad (4.15)$$

where Q_i^* and Q_j^* are the optimal order quantities for the coordinated scenarios.

Similar to the Coordinated Competition game, we obtain the following comparative statics:

Corollary 3. *At the SPNE of the Hybrid Competition game, the expected market supply is increasing in α_i and α_j . As a result, the expected market price is decreasing in α_i and α_j .*

Corollary 4. *At the SPNE of the Hybrid Competition game, the optimal order quantity and expected profit for the coordinated supply chain i (uncoordinated supply chain j) is decreasing in α_j (α_i) and increasing in α_i (α_j).*

Corollary 4 indicates that under a hybrid competition game, a supply chain's profit and order size increase if its supplier is more reliable and decrease as the competitor's supplier becomes more reliable.

Also note that compared with the Coordinated Competition Game, the order quantity for the coordinated supply chain i becomes larger as $Q_i^h(m^*) - Q_i^* = \frac{a\alpha_j(2-\alpha_i)}{4(4-\alpha_i\alpha_j)} > 0$ for any fixed α_i and α_j . However, the order quantity for the uncoordinated supply chain j becomes smaller; in fact it is reduced by half, $Q_j^h(m^*) = Q_j^*/2$. More detailed discussions on the comparison about expected market supplies and profits of both games will be given in Section 5.

4.3 Uncoordinated Competition Game

Assume that supply chain $i \in \{1, 2\}$ adopts a linear contract with wholesale price ω_i and penalty S_i for failed delivery. We use superscript "u" to indicate that both chains are uncoordinated. Define $m_i = \alpha_i \omega_i - (1 - \alpha_i) S_i$.

Definition 5 (Uncoordinated Competition Game). *An Uncoordinated Competition game is defined as*

- *Players: a set of four players: retailer i , retailer j , supplier i , and supplier j ,*
- *There are two stages in the game*
 - *Stage 1: two suppliers announce the contract term m_i and m_j simultaneously.*
 - *Stage 2: two retailers choose production quantity Q_i and Q_j simultaneously.*
- *Payoff: the expected revenues for the three players are $E(\pi_{i,r}(Q_i, Q_j, m_i))$, $E(\pi_{j,r}(Q_i, Q_j, m_j))$, $E(\pi_{i,s}(Q_i, m_i))$, and $E(\pi_{j,s}(Q_j, m_j))$, respectively. The payoff functions are defined similarly as in eqs. (4.4) and (4.5).*

The SPNE in our case can be characterized as follows.

Definition 6. *A pure strategy SPNE $(Q_i^u(m_i, m_j), Q_j^u(m_i, m_j), m_i^*, m_j^*)$ of the game between two uncoordinated supply chains satisfies:*

1. *Retailers compete with each other in quantities to maximize their profits for any given contracts (m_i, m_j) (not necessary optimal), i.e.,*

$$Q_i^u(m_i, m_j) \in \arg \max_{Q_i \geq 0} E(\pi_{i,r}(Q_i, Q_j^u(m_i, m_j), m_i)), \quad (4.16)$$

2. *Suppliers compete with each other in contract terms to maximize their profits given the choices of retailers, i.e.,*

$$m_i^* \in \arg \max_{m_i \geq 0} m_i Q_i^u(m_i, m_j^*), \quad (4.17)$$

The following lemma characterizes the equilibrium strategies of the retailers as functions of the contract terms m_i and m_j .

Lemma 2. *Let $g_i = (2 - \alpha_j)a + m_j - \frac{2m_i}{\alpha_i}$ and $g_j = (2 - \alpha_i)a + m_i - \frac{2m_j}{\alpha_j}$. At the SPNE of two uncoordinated supply chains,*

- i) *When $g_i > 0$ and $g_j > 0$,*

$$Q_i^u(m_i, m_j) = Q_i^* + \frac{\alpha_i m_j - 2m_i}{\alpha_i(4 - \alpha_i \alpha_j)} = \frac{g_i}{4 - \alpha_i \alpha_j},$$

ii) When $g_i \leq 0$ and $m_j < a\alpha_j$, $Q_i^u(m_i, m_j) = 0$ and

$$Q_j^u(m_i, m_j) = \frac{a}{2} - \frac{m_j}{2\alpha_j}.$$

iii) When $g_j \leq 0$ and $m_i < a\alpha_i$, $Q_j^u(m_i, m_j) = 0$ and

$$Q_i^u(m_i, m_j) = \frac{a}{2} - \frac{m_i}{2\alpha_i}.$$

iv) When $m_i \geq a\alpha_i$ and $m_j \geq a\alpha_j$, $Q_i^u(m_i, m_j) = Q_j^u(m_i, m_j) = 0$.

The four areas in Fig. 4 correspond to the four cases in Lemma 2, respectively. In area (I) both retailers choose to order positive amount since the both contracts are attractive (i.e., both suppliers charge small expected profit margins m_1 and m_1); in area (II) retailer 1 chooses not to order since m_1 is too high while retailer 2 orders; in area (III) retailer 2 chooses not to order since m_2 is too high while retailer i orders; in areas (IV) neither retailer choose to order since both m_1 and m_2 are very high.

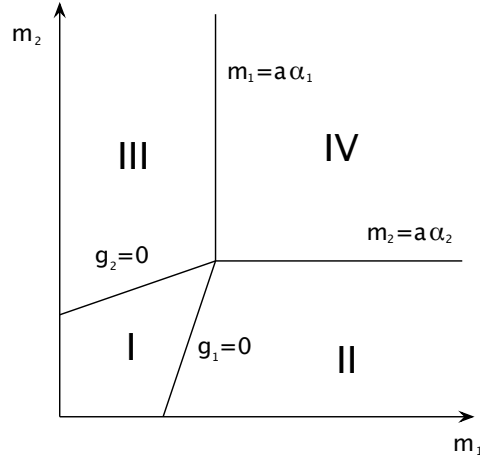


Figure 4: Choice of m_i and Equilibrium

Now, we are ready to derive the optimal wholesale-price contract terms.

Theorem 3. *At an SPNE of the Uncoordinated Competition game, supplier i chooses the contract terms ω_i^* and S_i^* such that*

$$m_i^* = \alpha_i \omega_i^* - (1 - \alpha_i) S_i^* = \frac{(2 - \alpha_i) a \alpha_i \alpha_j + 4 \alpha_i a (2 - \alpha_j)}{16 - \alpha_i \alpha_j}.$$

The order quantity of retailer i is

$$Q_i^u = Q_i^* + \frac{\alpha_i m_j - 2m_i}{\alpha_i(4 - \alpha_i \alpha_j)} = \frac{2a(8 - 2a_j - a_i a_j)}{(16 - \alpha_i \alpha_j)(4 - \alpha_i \alpha_j)}. \quad (4.18)$$

The expected profit of retailer i is

$$E(\pi_{i,r}^u) = \frac{4\alpha_i a^2 (8 - 2a_j - a_i a_j)^2}{(16 - \alpha_i \alpha_j)^2 (4 - \alpha_i \alpha_j)^2}$$

The expected profit of supplier i is

$$E(\pi_{i,s}^u) = m_i^* Q_i^u = \frac{2a^2 \alpha_i (8 - 2a_j - a_i a_j)^2}{(16 - \alpha_i \alpha_j)^2 (4 - \alpha_i \alpha_j)}.$$

The total expected profit of supply chain i is

$$E(\pi_i^u) = E(\pi_{i,s}^u) + E(\pi_{i,r}^u) = \frac{2a^2 \alpha_i (6 - \alpha_i \alpha_j) (8 - 2a_j - a_i a_j)^2}{(16 - \alpha_i \alpha_j)^2 (4 - \alpha_i \alpha_j)^2}. \quad (4.19)$$

Various quantities for supplier j and retailer j can be obtained similarly.

We obtain the following comparative statics:

Corollary 5. *At the SPNE of the Uncoordinated Competition game, the expected market supply is increasing in α_i and α_j . As a result, the expected market price is decreasing in α_i and α_j .*

Corollary 6. *At the SPNE of the Uncoordinated Competition game, the optimal order quantity for retailer i is decreasing in α_j and increasing in α_i . The expected total profit for supply chain i is decreasing in α_j and increasing in α_i .*

5. Strategic Decisions

In the previous section, we have derived the optimal contract design and inventory ordering decisions for the coordinated, hybrid and uncoordinated competition games. In this section we consider the strategic decision of which type of contract to choose.

First, we examine whether a supply chain should choose to coordinate or not. We compare supply chain's profits under the coordinated, hybrid, and uncoordinated competition games based on the results in Section 4. We observe that

- Between the hybrid and uncoordinated competition games, $E(\pi_i^h) > E(\pi_i^u)$.
- Between the hybrid and coordinated games, $E(\pi_i^h) > E(\pi_i^*)$ and $E(\pi_j^h) < E(\pi_j^*)$.

Hence we have the following result:

Theorem 4. *Given the competing supply chain's fixed strategic decision (coordinate or not to coordinate), a supply chain is always better off by choosing to coordinate. That is, coordination is a dominant strategy.*

A natural question to ask is whether each individual retailer and supplier in an uncoordinated chain have the right incentives to move into a coordinating contract.

Lemma 3. *For any given linear non-coordinating contracts, there always exists a coordinating revenue-sharing contract that makes both the supplier and retailer better off.*

Next, we examine customer service in terms of expected total market supply and price under the coordinated, the hybrid, and the uncoordinated competition games. From the customers' point of view, a higher total expected market supply means a lower expected market price, and thus a better customer service.

Theorem 5. *In terms of total expected market supply, the coordinated competition game provides the best service to customers, the hybrid competition game provides the middle level, and the uncoordinated competition game provides the worst service, i.e.,*

$$\alpha_i Q_i^* + \alpha_j Q_j^* > \alpha_i Q_i^h + \alpha_j Q_j^h > \alpha_i Q_i^u + \alpha_j Q_j^u. \quad (5.1)$$

Theorem 5 indicates that customers always benefit from coordination with more supplies and a lower price. To gain further insights into Theorem 5, we examine the special special case of symmetric supply chains, i.e., $\alpha_i = \alpha_j = \alpha$.

- Symmetric coordinated game: the optimal order size is

$$Q_i^* = \frac{a}{2 + \alpha}, \quad i = 1, 2,$$

Table 1: Order Quantity for Each Supply Chain
Supply Chain i

		uncoordinated	coordinated
Supply Chain j	uncoordinated	$\frac{2q}{4-\alpha}, \frac{2q}{4-\alpha}$	$\frac{(4+\alpha)q}{4}, \frac{q}{2}$
	coordinated	$\frac{q}{2}, \frac{(4+\alpha)q}{4}$	q, q

and the expected total market supply is

$$\alpha_1 Q_1^* + \alpha_2 Q_2^* = \frac{2a\alpha}{2+\alpha} \triangleq q.$$

- Symmetric uncoordinated game: the optimal order size is

$$Q_i^u = \frac{2a}{(4-\alpha)(2+\alpha)}, \quad i = 1, 2,$$

and the expected total market supply is

$$\alpha_1 Q_1^u + \alpha_2 Q_2^u = \frac{2q}{(4-\alpha)}.$$

- Symmetric hybrid game: the optimal order sizes for both chains are

$$\begin{aligned} Q_i^h &= \frac{a(8-2\alpha-\alpha^2)}{4(4-\alpha^2)} = \frac{a(4+\alpha)}{4(2+\alpha)} = \frac{(4+\alpha)q}{4} \quad (\text{coordinated one}), \\ Q_j^h &= \frac{a}{2(2+\alpha)} = \frac{q}{2} \quad (\text{un-coordinated one}). \end{aligned}$$

These results are summarized in Table 1. It can be seen that the coordinated competition game has the highest total expected order among all three scenarios, the hybrid competition game comes in the second, and the uncoordinated competition game comes in the third. This result suggests that coordination leads to the most benefits to the customers.

Finally, we show that between the coordinated and uncoordinated games, the revenue difference $E(\pi_i^*) - E(\pi_i^u)$ can be either negative or positive for both chains $i = 1, 2$.

Theorem 6. *Depending on the reliability of each supply chain, coordination efforts may or may not result in positive gains for each supply chain.*

To further understand the implications of Theorem 6, let us consider the special case with symmetric supply chains, i.e., $\alpha_i = \alpha_j = \alpha$. It can be verified that

Table 2: Payoff Table for Strategic Choice of Coordinating Supply Chain i

		uncoordinated	coordinated
Supply Chain j	uncoordinated	$\frac{(12-2\alpha^2)R}{(4-\alpha)^2}, \frac{(12-2\alpha^2)R}{(4-\alpha)^2}$	$\frac{(6-\alpha^2)R}{8}, \frac{(4+\alpha)^2R}{16}$
	coordinated	$\frac{(4+\alpha)^2R}{16}, \frac{(6-\alpha^2)R}{8}$	R, R

- Symmetric coordinated game:

$$E(\pi_i^*) = \frac{\alpha a^2}{(2+\alpha)^2} \triangleq R, \quad i = 1, 2$$

- Symmetric uncoordinated game:

$$E(\pi_i^u) = E(\pi^u) = \alpha \left(\frac{a}{2+\alpha} \right)^2 \left(\frac{12-2\alpha^2}{16-8\alpha+\alpha^2} \right) = \frac{(12-2\alpha^2)R}{(4-\alpha)^2}.$$

- Symmetric hybrid game: chain 1 is coordinated and chain 2 is not, thus

$$E(\pi_1^h) = \alpha \left(\frac{a}{2+\alpha} \right)^2 \left(\frac{8-2\alpha-\alpha^2}{8-4\alpha} \right)^2 = \frac{(4+\alpha)^2R}{16},$$

$$E(\pi_2^h) = \frac{(6-\alpha^2)R}{8}.$$

These results are summarized in Table 2. It can be verified that $\frac{(4+\alpha)^2}{16} > \frac{12-2\alpha^2}{(4-\alpha)^2}$, which implies that an uncoordinated supply chain is better off by moving toward coordination if the other chain remains uncoordinated (i.e., moving from the bottom left cell to the upper left cell). Furthermore, it can be seen that, $\frac{(6-\alpha^2)}{8} < 1$, which suggests that an uncoordinated supply chain is better off by moving toward coordination if the other chain is coordinated (i.e., moving from the upper right cell to the bottom right cell). Hence, as Theorem 4 suggested, coordination is the dominant strategy for both supply chains, meaning that both supply chains would like coordinate if they can have the flexibility.

It is interesting to note that when $\alpha < \frac{2}{3}$, we have $\frac{(12-2\alpha^2)}{(4-\alpha)^2} < 1$, i.e., the profit for each chain in the coordinated competition game is higher than that in the uncoordinated competition game (this is in contrast to the finding in Boyaci and Gallego 2004). Hence, if the supply chains are not very reliable, it is of the chain interests to first coordinate and then compete since coordination aligns

the intra-chain objectives and guarantees a profit gain in the long run. When $\alpha > \frac{2}{3}$, however, a chain receives less profit in the coordinated game compared with the uncoordinated game. This is the case of prisoner's dilemma, i.e., both supply chains are worse off under the coordinated scenario although coordination is the dominated strategy. In either case, customers benefit through the coordinations of supply chains.

6. Conclusion

In this paper, we provide a systematic examination on how to design and operate supply chains to effectively deal with supply uncertainty and competitions for short-life-cycle products. We derive the equilibrium decisions of ordering decisions and contract terms for the coordinated, hybrid, and uncoordinated competition games. By comparing these three scenarios, we obtain the following managerial insights on supply chain management with the presence of competition and supply uncertainty.

First, supply chain coordination is always a dominant strategy for each supply chain. The first mover in integrating decisions is likely to reap the benefits in aligning objectives within the supply chain. Given that one supply chain decides to coordinate, the other supply chain would also follow. In the long run, the coordinated scenario is the unique equilibrium.

Second, depending on the level of supply uncertainty, coordination efforts may or may not result in positive gains for each supply chain. For example, in the competition between two symmetric supply chains, coordination efforts guarantee a positive gain for each supply chain when the supply is highly uncertain. However, when the supply is sufficiently reliable, coordination results in lower profit for each supply chain. Nevertheless, coordination always benefits customers in terms of lower expected market price.

Third, for a supply chain with linear contract, there exists a coordinating contract which makes both supplier and retailer better off regardless whether the other supply chain coordinates or not.

The above findings are derived through closed-form analysis, facilitated by several simplifying assumptions. We may extend our analysis in several directions. First, we can consider contingent sourcing, i.e., after a retailer observes his supplier's status, he may purchase from another reliable

source or from the spot market. Second, we may explore the effect of correlated supply disruptions, i.e., a severe crisis could disrupt both chains simultaneously. Third, suppliers and retailers may have private information about demand and reliability. Incorporating these factors will add extra complexity into the analysis and make the closed-form results very tedious and difficult to obtain. Fourth, we may study different types of games (e.g., Bertrand competition), different type of contracts, or nonlinear pricing models and nonlinear cost functions. In this paper, we studied the revenue sharing contract and linear wholesale-price contract and assumed that supplier is the one who dictates the contract terms. It could be useful to consider other contract forms. Fifth, we can also consider the investment in reliability levels. We leave the study of these extensions to future research.

Appendix

Proof of Theorem 1. We shall first prove that both planners will have positive production quantities at the Nash equilibrium. According to the first order conditions, the Nash equilibrium (Q_i^*, Q_j^*) should satisfy

$$\alpha_i(a - 2Q_i^* - \alpha_j Q_j^*) \leq 0, \text{ with “=” if } Q_i^* > 0, \quad (6.1)$$

$$\alpha_j(a - 2Q_j^* - \alpha_i Q_i^*) \leq 0, \text{ with “=” if } Q_j^* > 0. \quad (6.2)$$

Since $a > 0$, it is clear that $(Q_i^*, Q_j^*) = (0, 0)$ does not satisfy eq. (6.1) and (6.2) simultaneously and is not an equilibrium.

Next, we show that $Q_i^* = 0$ and $Q_j^* > 0$ is not an equilibrium either. We prove this by contradiction. Assume that $Q_i^* = 0$ and $Q_j^* > 0$. Then eq. (6.1) can be simplified as

$$\alpha_i(a - \alpha_j Q_j^*) \leq 0,$$

meaning that $Q_j^* \geq \frac{a}{\alpha_j} > 0$. This means that eq. (6.2) is

$$\alpha_j(a - 2Q_j^*) = 0,$$

implying that $Q_j^* = \frac{a}{2} < \frac{a}{\alpha_j}$, which leads to a contradiction.

Hence, in the equilibrium, both (Q_i^*, Q_j^*) are positive, meaning that both the equality signs in eq. (6.1) and (6.2) hold. Since α_i and α_j are positive, we have

$$a - 2Q_i^* - \alpha_j Q_j^* = 0 \text{ and } a - 2Q_j^* - \alpha_i Q_i^* = 0.$$

Solving these two equations, we obtain the optimal production quantity Q_i^* as shown in (4.1).

From (4.1), we find that the optimal supply chain profit equals:

$$\begin{aligned} E(\pi_i^*) &= \alpha_i Q_i^* [\alpha_j (a - Q_i^* - Q_j^*) + (1 - \alpha_j)(a - Q_i^*)] \\ &= \frac{\alpha_i (2 - \alpha_j)^2 a^2}{(4 - \alpha_i \alpha_j)^2} = \alpha_i (Q_i^*)^2. \end{aligned} \quad (6.3)$$

■

Proof of Lemma 1. Since ϕ is a constant and $0 < \alpha_i \leq 1$, by looking at the best responses of both retailers we know that the following should be satisfied at an SPNE

$$a - 2Q_i^h(m) - \alpha_j Q_j^h(m) \leq 0, \quad (6.4)$$

$$\alpha_j (a - 2Q_j^h(m) - \alpha_i Q_i^h(m)) - m \leq 0. \quad (6.5)$$

Since $a > 0$, it is clear that $Q_i^h(m) = Q_j^h(m) = 0$ is not an equilibrium for any choice of m . We now discuss three cases.

Case 1: If retailer j chooses not to order and retailer i orders at the equilibrium, i.e., $Q_j^h(m) = 0$ and $Q_i^h(m) > 0$, eq. (6.4) and (6.5) become

$$a - 2Q_i^h(m) = 0 \text{ and } \alpha_j (a - \alpha_i Q_i^h(m)) - m \leq 0.$$

These two conditions are satisfied if and only if $m \geq \frac{a\alpha_j(2-\alpha_i)}{2}$. In other words, if the expected profit margin of supplier j exceeds the threshold $\frac{a\alpha_j(2-\alpha_i)}{2}$, then retailer j will not order.

Case 2: If retailer i chooses not to order and retailer j orders at the equilibrium, i.e., $Q_i^h(m) = 0$ and $Q_j^h(m) > 0$, eq. (6.4) and (6.5) become

$$a - \alpha_j Q_j^h(m) \leq 0 \text{ and } \alpha_j (a - 2Q_j^h(m)) - m = 0.$$

The above two conditions are satisfied if and only if $m \leq a(\alpha_j - 2)$. Since $m \geq 0$ but $\alpha_j - 2 < 0$, these conditions will not be satisfied.

Case 3: If both retailers order at the equilibrium, i.e., both $Q_i^h(m)$ and $Q_j^h(m)$ are positive. Eq. (6.4) and (6.5) become

$$a - 2Q_i^h(m) - \alpha_j Q_j^h(m) = 0 \text{ and } a - 2Q_j^h(m) - \alpha_i Q_i^h(m) - \frac{m}{\alpha_j} = 0.$$

We find that $Q_i^h(m)$ and $Q_j^h(m)$ as shown in (4.9) and (4.10) respectively. In this case we can derive that $0 \leq m < \frac{a\alpha_j(2-\alpha_i)}{2}$. ■

Proof of Theorem 2. First, we analyze how the supplier j will choose m^* . It is obvious that the optimal m^* for supplier j must satisfy $a(\alpha_j - 2) < m^* < \frac{a\alpha_j(2-\alpha_i)}{2}$, otherwise, supplier j either earns zero profit (when $m \geq \frac{a\alpha_j(2-\alpha_i)}{2}$) or negative profit (when $m \leq a(\alpha_j - 2) < 0$). According to Lemma 1, with $a(\alpha_j - 2) < m < \frac{a\alpha_j(2-\alpha_i)}{2}$, the retailer's equilibrium order is

$$Q_j^h(m) = \frac{a\alpha_j(2-\alpha_i) - 2m}{\alpha_j(4-\alpha_i\alpha_j)}.$$

Hence, the profit function for supplier j is

$$E\left(\pi_{j,s}(Q_j^h(m), m)\right) = mQ_j^h(m) = m\left(\frac{a\alpha_j(2-\alpha_i) - 2m}{\alpha_j(4-\alpha_i\alpha_j)}\right).$$

It can be verified that the supplier j 's profit function is concave with respect to m . The first-order condition yields

$$a\alpha_j(2-\alpha_i) - 4m^* = 0,$$

which leads to

$$m^* = \frac{a\alpha_j(2-\alpha_i)}{4} = \alpha_j\omega^* - (1-\alpha_j)S^*.$$

In other words, to maximize the expected profit, supplier j shall choose the contract terms ω^* and S^* such that eq. (4.11) is satisfied. Note that there are many pairs of ω and S that can satisfy this condition.

Next, we substitute eq. (4.11) into eqs. (4.9) and (4.10) to obtain eqs. (4.12) and (4.13).

The optimal expected profit for supply chain i is

$$\begin{aligned} E(\pi_i^h) &= \alpha_i Q_i^h(m^*) \left(a - Q_i^h(m^*) - \alpha_j Q_j^h(m^*) \right) \\ &= \frac{\alpha_i a (8 - 2\alpha_j - \alpha_i \alpha_j)}{4(4 - \alpha_i \alpha_j)} \left(a - \frac{a(8 - 2\alpha_j - \alpha_i \alpha_j)}{4(4 - \alpha_i \alpha_j)} - \frac{\alpha_j a (2 - \alpha_i)}{2(4 - \alpha_i \alpha_j)} \right) \\ &= \alpha_i (Q_i^*)^2 \left(\frac{8 - 2\alpha_j - \alpha_i \alpha_j}{8 - 4\alpha_j} \right)^2. \end{aligned} \tag{6.6}$$

and that for supply chain j is

$$\begin{aligned}
E(\pi_j^h) &= \alpha_j Q_j^h(m^*)(a - Q_j^h(m^*) - \alpha_i Q_i^h(m^*)) \\
&= \frac{\alpha_j a(2 - \alpha_i)}{2(4 - \alpha_i \alpha_j)} \left(a - \frac{a(2 - \alpha_i)}{2(4 - \alpha_i \alpha_j)} - \frac{\alpha_i a(8 - 2\alpha_j - \alpha_i \alpha_j)}{4(4 - \alpha_i \alpha_j)} \right) \\
&= \alpha_j (Q_j^*)^2 \left(\frac{6 - \alpha_i \alpha_j}{8} \right), \tag{6.7}
\end{aligned}$$

where Q_i^* and Q_j^* are the optimal order quantities for the coordinated scenarios. ■

Proof of Theorem 3. The profit for supplier i is

$$E(\pi_{i,s}^u) = m_i Q_i^u(m_i, m_j).$$

The best response for supplier i and j are respectively

$$\begin{aligned}
\frac{\partial E(\pi_{i,s}^u)}{\partial m_i} &= \frac{(2 - \alpha_j)a + m_j - \frac{4m_i}{\alpha_i}}{4 - \alpha_i \alpha_j} = 0 \\
\frac{\partial E(\pi_{j,s}^u)}{\partial m_j} &= \frac{(2 - \alpha_i)a + m_i - \frac{4m_j}{\alpha_j}}{4 - \alpha_i \alpha_j} = 0.
\end{aligned}$$

Solving these two equations, we obtain the result in Theorem 3. ■

Proof of Lemma 2. First, we write down the expected profit of retailer i as the following.

$$E(\pi_{i,r}(Q_i, Q_j, m_i, m_j)) = \alpha_i Q_i(a - Q_i - \alpha_j Q_j) - m_i Q_i. \tag{6.8}$$

We suppress the notation on (m_i, m_j) whenever it is convenient. By looking at the best responses of both retailers we know that the following should be satisfied at an SPNE

$$\alpha_i(a - 2Q_i^u - \alpha_j Q_j^u) - m_i \leq 0, \tag{6.9}$$

$$\alpha_j(a - 2Q_j^u - \alpha_i Q_i^u) - m_j \leq 0. \tag{6.10}$$

We now discuss four cases by referring to Fig. 4.

From Fig. 4, we can see that the first quadrant of (m_i, m_j) plane can be divided into four areas, where area (I) corresponds to the case (i) in Lemma 2, area (II) corresponds to the case (ii), area (III) corresponds to the case (iii), and areas (IV), (V), and (VI) corresponds to the case (iv).

Case i: If both retailers choose to order, i.e., $Q_i^u > 0$, eq. (6.9) and (6.10) become

$$\alpha_i(a - 2Q_i^u - \alpha_j Q_j^u) - m_i = 0 \text{ and } \alpha_j(a - 2Q_j^u - \alpha_i Q_i^u) - m_j = 0.$$

These two conditions are satisfied if and only if $g_i > 0$ and $g_j > 0$.

Case ii: If retailer i chooses not to order and retailer j orders at the equilibrium, i.e., $Q_i^u = 0$ and $Q_j^u > 0$, eq. (6.9) and (6.10) become

$$\alpha_i(a - \alpha_j Q_j^u) - m_i \leq 0 \text{ and } \alpha_j(a - 2Q_j^u) - m_j = 0.$$

The above two conditions are satisfied if and only if $g_i \leq 0$ and $m_j < a\alpha_j$.

Case iii: If retailer j chooses not to order and retailer i orders at the equilibrium, i.e., $Q_i^u > 0$ and $Q_j^u = 0$, eq. (6.9) and (6.10) become

$$\alpha_i(a - 2Q_i^u) - m_i = 0 \text{ and } \alpha_j(a - \alpha_i Q_i^u) - m_j \leq 0.$$

The above two conditions are satisfied if and only if $g_j \leq 0$ and $m_i < a\alpha_i$.

Case iv: If retailer j chooses not to order and retailer i orders at the equilibrium, i.e., $Q_i^u > 0$ and $Q_j^u = 0$, eq. (6.9) and (6.10) become

$$\alpha_i a - m_i \leq 0 \text{ and } \alpha_j a - m_j \leq 0.$$

The above two conditions are satisfied if and only if $m_i \geq a\alpha_i$ and $m_j \geq a\alpha_j$. ■

Proof of Theorem 5. From eqs (4.1), (4.12), (4.13), and (4.18), we find

$$\begin{aligned} \alpha_i Q_i^* + \alpha_j Q_j^* &= \frac{a\alpha_i(2 - \alpha_j) + a\alpha_j(2 - \alpha_i)}{4 - \alpha_i\alpha_j}, \\ \alpha_i Q_i^h + \alpha_j Q_j^h &= \frac{a\alpha_i(8 - 2\alpha_j - \alpha_i\alpha_j) + 2a\alpha_j(2 - \alpha_i)}{4(4 - \alpha_i\alpha_j)}, \text{ and} \\ \alpha_i Q_i^u + \alpha_j Q_j^u &= \frac{2a\alpha_i(8 - 2\alpha_j - \alpha_i\alpha_j) + 2a\alpha_j(8 - 2\alpha_i - \alpha_i\alpha_j)}{(16 - \alpha_i\alpha_j)(4 - \alpha_i\alpha_j)}. \end{aligned}$$

It can be verified that

$$\alpha_i Q_i^* + \alpha_j Q_j^* - (\alpha_i Q_i^h + \alpha_j Q_j^h) = \frac{a\alpha_j(2 - \alpha_i)^2}{4(4 - \alpha_i\alpha_j)} > 0.$$

And

$$\begin{aligned} \alpha_i Q_i^h + \alpha_j Q_j^h - (\alpha_i Q_i^u + \alpha_j Q_j^u) &= \frac{\alpha_i(64 - 32\alpha_j - 16\alpha_i\alpha_j + 4\alpha_i\alpha_j + \alpha_i^2\alpha_j + 4\alpha_j)}{4(16 - \alpha_i\alpha_j)(4 - \alpha_i\alpha_j)} \\ &> \frac{\alpha_i(16 + 4\alpha_i\alpha_j + \alpha_i^2\alpha_j + 4\alpha_j)}{4(16 - \alpha_i\alpha_j)(4 - \alpha_i\alpha_j)} > 0. \end{aligned}$$

Hence, $\alpha_i Q_i^* + \alpha_j Q_j^* > \alpha_i Q_i^h + \alpha_j Q_j^h > \alpha_i Q_i^u + \alpha_j Q_j^u$. ■

Proof of Theorem 4. From eqs (4.14) and (4.19), we see that

$$E(\pi_i^h) - E(\pi_i^u) = E(\pi_i^u) \left[\frac{(16 - \alpha_i \alpha_j)^2}{16(12 - 2\alpha_i \alpha_j)} - 1 \right] = E(\pi_i^u) \left[\frac{64 + (\alpha_i \alpha_j)^2}{16(12 - 2\alpha_i \alpha_j)} \right] > 0,$$

which implies that an uncoordinated chain is better off by moving toward coordination if the other supply chain is not coordinated.

From eqs. (4.2) and (4.14), we see that

$$E(\pi_i^h) = E(\pi_i^*) \left(\frac{8 - 2\alpha_j - \alpha_i \alpha_j}{4(2 - \alpha_j)} \right)^2 > E(\pi_i^*),$$

since $\frac{8 - 2\alpha_j - \alpha_i \alpha_j}{8 - 4\alpha_j} > 1$.

From eq. (4.15), we see that

$$E(\pi_j^h) = E(\pi_j^*) \left(\frac{6 - \alpha_i \alpha_j}{8} \right) < E(\pi_j^*),$$

since $\frac{6 - \alpha_i \alpha_j}{8} < 1$. This implies that an uncoordinated supply chain is better off by coordinating if the other supply chain is coordinated.

Hence, coordination is the dominant strategy regardless whether the other supply chain is coordinated or not.

Next, from eq. (4.19) and (4.14), we see that

$$E(\pi_i^u) - E(\pi_i^*) = E(\pi_i^*) \left[\frac{(12 - 2\alpha_i \alpha_j)(8 - 2\alpha_i - \alpha_i \alpha_j)^2}{(16 - \alpha_i \alpha_j)^2 (2 - \alpha_j)^2} - 1 \right].$$

But the sign of the term in the bracket is indefinite. ■

Proof of Collary 3. We examine the coordination decision for a supply chain in two cases: (i) when the other supply chain is coordinated; and (ii) when the other supply chain is uncoordinated.

Case (i) when the other supply chain is coordinated: Without loss of generality, we assume that supply chain j uses a linear contract and supply chain i is coordinated. As Theorem 4 suggested, $E(\pi_j^h) < E(\pi_j^*)$, meaning that the entire supply chain j would be better off if it coordinates. Suppose that under a given linear contract, retailer j earns a profit of $E(\pi_{j,r}^h)$ and supplier j earns a profit of $E(\pi_{j,s}^h)$. Consider when supplier j and retailer j engage in a revenue-sharing contract

with profit split ratio $\phi_i = E(\pi_{j,r}^h)/E(\pi_j^h)$. It is clear that retailer j is better off since

$$\phi_i E(\pi_j^*) > \phi_i E(\pi_j^h) = \frac{E(\pi_{j,r}^h)}{E(\pi_j^h)} E(\pi_j^h) = E(\pi_{j,r}^h).$$

and so is supplier j since

$$(1 - \phi_i)E(\pi_j^*) > (1 - \phi_i)E(\pi_j^h) = E(\pi_{j,s}^h).$$

Case (ii) when the other supply chain is uncoordinated: Without loss of generality, we assume that supply chain j uses a linear contract and supply chain i can choose either to coordinate or stick with a given linear contract. As Theorem 4 suggested, $E(\pi_i^u) < E(\pi_i^h)$. We define $\phi_i = E(\pi_{i,r}^u)/E(\pi_i^u)$ and let ϕ_i be the profit split ratio of a revenue-sharing contract, we see that both the retailer and supplier are better off under this contract.

In conclusion, there exists a coordinating contract that achieves Pareto improvement in a supply chain regardless whether the other supply chain coordinates or not. ■

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