# Contract-Based Cooperative Spectrum Sharing

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***Abstract*—Providing proper economic incentives is essential for the success of dynamic spectrum sharing. Cooperative spectrum sharing is one effective way to achieve this goal. In cooperative spectrum sharing, secondary users (SUs) relay traffics for primary users (PUs), in exchange for dedicated transmission time for the SUs' own communication needs. In this paper, we study the cooperative spectrum sharing under incomplete information, where SUs' types (which capture the relay channel gains and the SUs' power costs) are private information and are not known to the PU. Inspired by the contract theory, we model the network as a labor market. The PU is an employer who offers a contract to the SUs. The contract consists of a set of items representing combinations of spectrum access time (i.e., reward) and relay power (i.e., contribution). The SUs are employees, and each of them selects the best contract item to maximize its payoff. We study the optimal contract design for both weakly and strongly incomplete information scenarios. First, we provide necessary and sufficient conditions for feasible contracts in both scenarios. In the weakly incomplete information scenario, we further derive the optimal contract that achieves the same maximum PU's utility as in the complete information benchmark. In the strongly incomplete information scenario, we propose a Decompose-and-Compare algorithm that achieves a close-to-optimal contract. We further show that the PU's expected utility loss due to the suboptimal algorithm and the strongly incomplete information are both relatively small (less than $2\%$ and $1.3\%$, respectively, in our numerical results with two SU types).**

## I. Introduction

With the explosive development of wireless services and networks, spectrum is becoming more congested and scarce. Dynamic spectrum sharing is a promising approach to increase spectrum efficiency and alleviate spectrum scarcity, as it enables the unlicensed secondary users (SUs) to dynamically access the spectrum licensed to the primary users (PUs) [1], [2]. The successful implementation of dynamic spectrum sharing requires many innovations in technology, economics, and policy. In particular, it is important to design the sharing mechanism such that PUs have incentive to open their licensed spectrum for sharing, and SUs have incentive to utilize the new spectrum opportunities despite of the potential costs.

Market-driven spectrum trading is a promising paradigm to address the incentive issue in dynamic spectrum sharing. With spectrum trading, PUs temporarily *sell* the spectrum to SUs to obtain either a monetary reward or a performance improvement. A particularly interesting trading scheme is *cooperative spectrum sharing*, where SUs relay traffics for PUs in order to get their own share of spectrum. A brief illustration of cooperative spectrum sharing is shown in Figure 1 on the next page. The SUs' transmitters ($\text{ST}_1 \sim \text{ST}_3$) act as cooperative relays for the PU in Phase I and Phase II, and transmit their own data in Phase III. The cooperative spectrum sharing leads to a win-win outcome for both PUs and SUs.

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The study of cooperative spectrum sharing mechanisms have only started recently [15]–[18]. The prior results all assumed complete network information, i.e., PUs know SUs' channel conditions, resource constraints, and costs of transmission powers. This assumption is often too strong for practical networks. In this paper, we study the cooperative spectrum sharing under *incomplete information*. We consider the general case that SUs have different *types* based on their relay channel gains and evaluations of power consumptions. The types are private information, and only an SU knows its own type. The only related paper that also deals with incomplete information is [19], which studied the interactions between a single PU and a single SU. We consider the interaction between a single PU and multiple SUs in the network.

To tackle this problem, we propose a contract-based cooperative spectrum sharing mechanism. Contract theory is an effective tool in designing the incentive compatible mechanism in a monopoly market under incomplete information [22]. The key idea is to offer the right contract items so that all agents have the incentive to truthfully reveal their private information. For the spectrum sharing problem, we can imagine the network as a labor market. The PU is an employer who offers a *contract* to the SUs. The contact consists of a set of contract items, which are combinations of spectrum access time (i.e., reward) and relay power (i.e., contribution). The SUs are employees, and each SU selects the best contract item according to its type. *We want to design an optimal contract that maximizes the PU's utility (average data rate) under the incomplete information of SUs' types*.

The main contributions of this paper are as follows:

- *New modeling and solution technique*: As far as we know, this is the first paper that tackles cooperative spectrum sharing under incomplete information based on contract theory.
- *Multiple information scenarios*: We study the issue of optimal contract design for three different scenarios: complete information (benchmark), weakly incomplete information, and strongly incomplete information. For the latter two scenarios, we characterize the necessary and sufficient conditions under which a contract is feasible.
- *Optimal contract design*: In the weakly incomplete information scenario, we derive the optimal contract that achieves the same maximum PU's utility as in the complete information benchmark. In the strongly incomplete information scenario, we propose a Decompose-

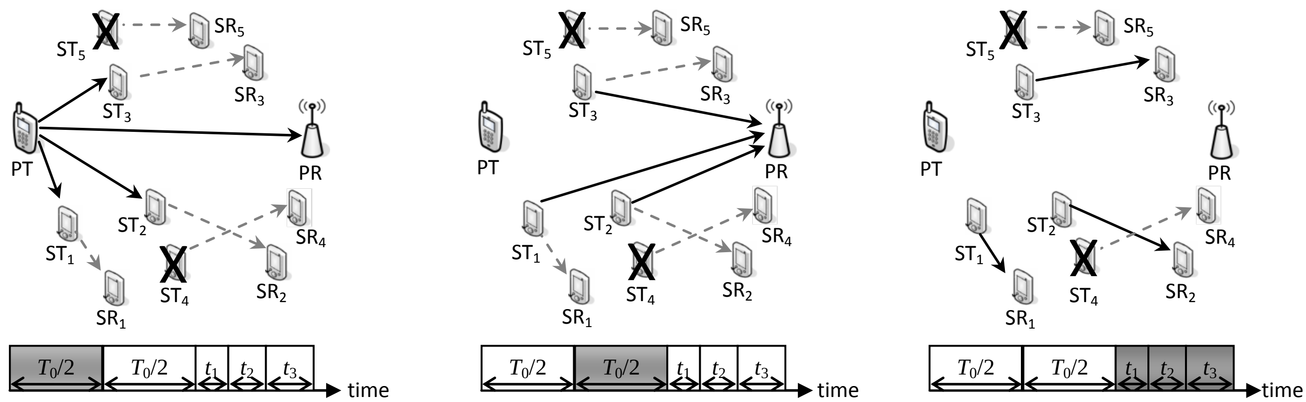


Fig. 1. Cooperative spectrum sharing with three phases in each time slot

TABLE I
FEASIBILITY CONDITIONS AND OPTIMALITY

| Network Information | Optimality | Sections in This Paper |
|---|---|---|
| **Complete (benchmark)** | Optimal | III |
| **Weakly Incomplete** | Optimal | IV and V |
| **Strongly Incomplete** | Close-to-Optimal | IV and VI |

and-Compare algorithm that obtains a close-to-optimal contract for the PU.

- *Performance analysis*: In the strongly incomplete information scenario, we quantify the PU's expected utility loss due to the suboptimal algorithm (by comparing it with the exhaustive search method) as well as the strongly incomplete information (by comparing it with the complete information benchmark). Both kinds of losses are are relatively small, i.e., less than $2\%$ and $1.3\%$ in our numerical results with two SU types, respectively.

The key results and the corresponding section numbers in this paper are summarized in Table I.

The rest of this paper is organized as follows. In Section II, we provide the system model and problem formulation. In Section III, we propose the optimal contract under complete information. In Section IV, we propose the necessary and sufficient conditions for feasible contracts under incomplete information. In Sections V and VI, we derive the optimal and close-to-optimal contracts in weakly and strongly incomplete information scenarios, respectively. We present the numerical results in Section VII. We review the related literatures in Section VIII and finally conclude in Section IX.

## II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a cognitive radio network with a primary licensed user (PU) and multiple secondary unlicensed users (SUs) as shown in Fig. 1. Each user is a dedicated transmitter-receiver pair. The PU has the exclusive usage right of the licensed spectrum band, but its transmission suffers from the poor channel condition between its transmitter $PT$ and receiver $PR$. We represent $M$ SUs by distinct transmitter-receiver pairs $\{ST_k - SR_k\}_{k=1}^{M}$. Each SU wants to have dedicated time to access the licensed band and transmit its own data. The PU can employ a subset of or all SUs to relay its traffic; the *involved* SUs will obtain dedicated transmission time for their own data. The interaction between the PU and the SUs involves three phases as in Fig. 1: Phases I and II for the cooperative communications with a total fixed length of time $T_0$,[1] and Phase III for the SUs' own transmissions. More specifically, we have

- Phase I: In the first half of the cooperative communication period ($T_0/2$), the primary transmitter $PT$ broadcasts its data to the primary receiver $PR$ and the involved SUs' transmitters (e.g., $ST_1$, $ST_2$, and $ST_3$ in Fig. 1). Note that SU 4 and SU 5 are not involved in this example.
- Phase II: In the remaining half of the cooperative communication period ($T_0/2$), the involved SUs' transmitters ($ST$s) decode the data received in Phase I and forward to the primary receiver $PR$ simultaneously using the space-time codes assigned by PU.[2] Through proper choice of space-time codes, SUs' simultaneous relay signals do not interference with each other at the primary receiver $PR$ [20].
- Phase III: PU rewards each involved SU with a dedicated time allocation for that SU's own data (e.g., $\{t_k\}_{k=1}^{3}$ for three involved SUs). SUs access the spectrum using TDMA and do not interfere with each other.

Here we assume that each involved SU can successfully decode PU's data in the first phase of cooperative communications. Thus we can focus on the relay links between the $ST$s and $PR$, which are the performance bottleneck of cooperative communications. This assumption can be relaxed if the PU can perform an initial screening over all SUs as follows. The PU first broadcasts a pilot signal to all SUs, and only those SUs replying correctly can choose to accept the contract and involve in the cooperative communications later. Notice that the involved SUs are often the ones that are close to the primary transmitter $PT$, and thus are not close to the primary

[1]The time period $T_0$ is a constant and is determined by PU's MAC layer and Physical layer specifications.

[2]We will discuss the details of the coordination between PU and SUs in Section II-C.

receiver $PR$. This explains why a $PT-ST$ channel is often better than the corresponding $ST-PR$ channel.

The PU and SUs have conflicting objectives in the above interactions. The PU wants the SUs to relay its traffic with high power levels, which will increase the PU's data rate but reduce the SUs' battery levels. An SU $k$ wants to obtain a large dedicated transmission time $t_k$, which will increase the SU's own performance but reduce the PU's utility (time average data rate). In Sections II-A and II-B, we will explain in details how the PU and SUs evaluate the trade-off between relay powers and time allocations. In Section II-C, we propose a contract-based framework which brings the PU and SUs together and resolve the conflicts.

### A. Primary User Model

In this subsection, we discuss how the PU evaluates relay powers and time allocations.

We first derive the PU's achievable data rate during the cooperative communications (i.e., Phases I and II in Fig. 1). Let us denote the set of involved SUs as $\mathcal{N}$ (e.g., $\mathcal{N}=\{1,2,3\}$ in Fig. 1). The received power (at the primary receiver $PR$) from SU $k$ is $p_k$, and the time allocation to this SU is $t_k$. *Without loss of generality, we normalize $T_0$ to be 1 in the rest of the paper.* Then $t_k$ can be viewed as the ratio of $t_k/T_0$.

- In Phase I, PU's transmitter broadcasts its data, and PU's receiver achieves a data rate (per unit time) of
$$R^{dir}=\log(1+\texttt{SNR}_{PT,PR}). \tag{1}$$
Note that $R^{dir}$ remains as a constant throughout the analysis.
- In Phase II, each involved SU successfully decodes PU's data and forwards to PU's receiver.

Thus the PU's total transmission rate during the cooperative communications (Phases I and II) is ([20])

$$\begin{aligned} r_{PU}^{relay} &= \frac{R^{dir}}{2}+\frac{1}{2}\log\left(1+\sum_{k\in\mathcal{N}}\texttt{SNR}_{ST_k,PR}\right) \\ &= \frac{R^{dir}}{2}+\frac{1}{2}\log\left(1+\frac{\sum_{k\in\mathcal{N}}p_k}{n_0}\right), \end{aligned} \tag{2}$$

where $n_0$ is the noise power, and constant $1/2$ is due to equal partition of the cooperative communication time into Phase I and Phase II. We can think (2) as the sum of transmission rates of two "parallel" channels, one from the $PT$ to $PR$ and the other from the set of involved SUs' transmitters to $PR$.

Based on the above discussion, we next compute the PU's average data rate during the entire time period (i.e., Phases I, II, and III). The cooperative communications only utilizes $1/(1+\sum_{k\in\mathcal{N}}t_k)$ fraction of the entire time period. The PU's objective is to maximize its **utility** (i.e., average transmission rate during the entire time period) as follows

$$\begin{aligned} u_{PU} &= \frac{1}{1+\sum_{k\in\mathcal{N}}t_k} r_{PU}^{relay} \\ &= \frac{1}{1+\sum_{k\in\mathcal{N}}t_k}\left(\frac{R^{dir}}{2}+\frac{1}{2}\log\left(1+\frac{\sum_{k\in\mathcal{N}}p_k}{n_0}\right)\right), \end{aligned} \tag{3}$$

which is decreasing in total time allocations to SUs (i.e., $\sum_{k\in\mathcal{N}}t_k$), and is increasing in the total received power from SUs (i.e., $\sum_{k\in\mathcal{N}}p_k$).

We want to emphasize that the utility calculation in (3) assumes that the PU involves at least one SU in the cooperative communication. The PU can also choose to have direct transmissions only in both Phases I and II and not to interact with the SUs (and thus there is no Phase III). The total rate in this direct transmission only approach is $R^{dir}$. This means that the PU will not choose cooperative communications if the utility in (3) is smaller than $R^{dir}$. In the rest of the analysis, we assume that $R^{dir}$ is small such that the PU wants to use cooperative communications. In Section VII, we will further explain what will happen when this is not true.

### B. Secondary User Model

Next we discuss how SUs evaluate relay powers $\{p_k\}_{k\in\mathcal{N}}$ and time allocations $\{t_k\}_{k\in\mathcal{N}}$. *We want to emphasize that the relay power is measured at the PU's receiver, not at the SUs' transmitters.* We consider a general model where SUs are heterogeneous in three aspects:

- SUs have different relay channel gains between their transmitters and the PU's receiver ($h_{ST,PR}$). If an SU $k$ wants to reach a received power level $p_k$ at the PU's receiver, it needs to transmit with a power $p_k^t=p_k/h_{ST_k,PR}$.
- SUs can achieve different (fixed) rates to their own receivers (i.e., data rate $r_{SU_k}$ over link $ST_k-SR_k$) with different (fixed) transmission power (i.e., $p_{SU_k}^t$).
- SUs have different cost $C_k$ per unit transmission power.

Note that the parameters $C_k$, $r_{SU_k}$, $p_{SU_k}^t$, and $h_{ST_k,PR}$ are SU $k$'s private information and are only known to himself.

We define an $SU\ k$'s **payoff** as $\pi_{SU_k}$, which is the difference between its own transmitted data during time allocation $t_k$ in Phase III and its cost of power consumption during Phase II and Phase III. That is,

$$\pi_{SU_k}=t_k r_{SU_k}-\left(t_k p_{SU_k}^t+\frac{1}{2}\frac{p_k}{h_{ST_k,PR}}\right)C_k. \tag{4}$$

We assume that every SU wants to have positive transmission time if it does not need to relay the PU's traffic, i.e., $r_{SU_k}-p_{SU_k}^t C_k\geq 0$ for all $k\in\mathcal{N}$. If this is not true for an SU, we can simply eliminate it from the network. Notice that (4) is increasing in time allocation $t_k$, but is decreasing in relay power $p_k$.

We can further simplify (4) by multiplying both sides by $2h_{ST_k,PR}/C_k$, which leads to the **normalized payoff**

$$\tilde{\pi}_{SU_k}:=\pi_{SU_k}\frac{2h_{ST_k,PR}}{C_k}=\frac{2h_{ST_k,PR}(r_{SU_k}-C_k p_{SU_k}^t)}{C_k}t_k-p_k. \tag{5}$$

Such normalization does not affect SUs' choice among different relay powers and time allocations. Thus it will not affect the contract design introduced later.

To facilitate later discussions, we define an $SU\ k$'s **type** as

$$\theta_k:=\frac{2h_{ST_k,PR}(r_{SU_k}-C_k p_{SU_k})}{C_k}>0, \tag{6}$$

which captures all private information of this SU. A large type $\theta_k$ means that the SU's own transmission is efficient (a large $r_{SU_k}$ or a small $p_{SU_k}$), or it has a good channel gain $h_{ST_k-PR}$ over the relay link, or it has a more efficient battery technology (a small $C_k$). With (6), we can simplify $SU$'s normalized payoff in (5) as

$$\pi_k(p_k, t_k) := \tilde{\pi}_{SU_k} = \theta_k t_k - p_k. \quad (7)$$

which is decreasing in PU's received power $p_k$ and increasing in PU's time allocation $t_k$.

Since each SU is selfish, a type-$\theta_k$ SU wants to choose relay power and time allocation to maximize its payoff in (7). Notice that an SU can always choose not to help the PU, which leads a zero payoff.

### C. Contract Formulation under Incomplete Information

After introducing PU's utility in (3) and SUs' (normalized) payoffs in (7), we are ready to introduce the contract mechanism that resolves the conflicting objectives between the PU and SUs.

Contract theory studies how economic decision-makers construct contractual arrangements, generally in the presence of asymmetric (private) information [22]. In our case, the SUs' types are private information and only known to themselves. The PU does not know the type of each SU, and needs to design a contract to attract the SUs to participate in cooperative communications.

To better understand the contract design in this paper, we can imagine the PU as an employer and SUs as employees in a labor market. The employer determines the contract, which specifies the relationship between the employee's performance (i.e., received relay powers) and reward (i.e., time allocation). If we denote $\mathcal{P}$ as the set of all possible relay powers and $\mathcal{T}$ as the set of all possible time allocations, then the contract specifies a $t \in \mathcal{T}$ for every $p \in \mathcal{P}$. Each distinct power-time association becomes a contract item. Once a contract is given, each SU will choose the contract item that maximizes its payoff in (7). The PU wants to optimize the contract items to maximize its utility in (3).

We consider $K$ types of SUs, with types denoted by the set $\Theta = \{\theta_1, \theta_2, ..., \theta_K\}$. Without loss of generality, we assume that $\theta_1 < \theta_2 < ... < \theta_K$. The total number of SUs in type-$\theta_k$ is $N_k$. According to the revelation principle [21], it is enough to consider the class of contract that enables the SUs to truthfully reveal their types. Because of this, it is enough to design a contract that consists $K$ contract items, one for each type.[3] We denote contract item intended for type-$\theta_k$ by $(p_k, t_k)$. The contract can be written as $\Phi = \{(p_k, t_k), \forall k \in \mathcal{K}\}$, where $\mathcal{K} = \{1, 2, ..., K\}$.

We will consider the optimal contract design for three information scenarios.

- *Complete information in Section III:* This is a benchmark case, where the PU knows each SU's type. We will compute the maximum utility that the PU can achieve in this case, which serves as an upper-bound of the PU's achievable utility in the incomplete information scenarios.
- *Weakly incomplete information in Section V:* The PU does not know each SU's type, but has knowledge of the set of types and the number of each type SUs in the market (i.e., $N_k$ for each type-$\theta_k$). We will show that the optimal contract in this case achieves the same PU's utility as in the complete information benchmark.
- *Strongly incomplete information in Section VI:* The PU only knows the total number of SUs ($N$) and the distribution of each type, but does not know the number of each type ($N_k$). The PU needs to design a contract to maximize its *expected* utility.

[3]Note that the contract items intended for different types may be same.

Once the PU has determined the contract, the interactions between the PU and SUs will follow four steps.

1) The PU broadcasts the contract $\Phi = \{(p_k, t_k), \forall k \in \mathcal{K}\}$ to all SUs.
2) After receiving the contract, each SU chooses one contract item that maximizes its payoff and informs the PU its choice.
3) After receiving all SUs confirmations, the PU informs the involved SUs (i.e., those choosing positive contract items) the space-time codes to use in Phase II and the transmission schedule in Phase III. Note that the length for transmission time for each involved SU is specified by the contract item and the PU can no longer change.
4) The communications start by following three phases in Fig. 1.

## III. Optimal Contract Design under complete information: The Benchmark Scenario

In the complete information scenario, the PU knows precisely the type of each SU. We will use the maximum PU's utility in this case to evaluate the contract performance in Sections V and VI. Without loss of generality, we assume that $N_k \geq 1$ for all $k \in \mathcal{K}$.

As the PU knows each SU's type, it can monitor and make sure that each SU accepts only the contract item designed for its type. The PU needs to ensure that the SUs have non-negative payoffs so that they are willing to accept the contract. In other words, the contract needs to satisfy the following individual rationality constraint.

*Definition 1:* (IR: Individual Rationality): A contract satisfies the individual rationality constraint if each type-$\theta_k$ SU receives a non-negative payoff by accepting the contract item for $\theta_k$, i.e.,

$$\theta_k t_k - p_k \geq 0, \quad \forall k \in \mathcal{K}. \quad (8)$$

We say a contract is *optimal* if it yields the maximum utility for the PU under the current information scenario. Different information scenarios may lead to different optimal contracts.

In the complete information scenario, an optimal contract maximizes the PU's utility as follows

$$\max_{\{(p_k,t_k)\geq \mathbf{0}, \forall k\}} \frac{\frac{R^{dir}}{2} + \frac{1}{2}\log\left(1 + \frac{\sum_{k\in\mathcal{K}} N_k p_k}{n_0}\right)}{1 + \sum_{k\in\mathcal{K}} N_k t_k}, \quad (9)$$
$$\text{subject to IR Constraints in Eq. (8).}$$

In this paper, the vector operations are component-wise (e.g, $(p_k, t_k) \geq \mathbf{0}$ means that $p_k \geq 0$ and $t_k \geq 0$) unless specified otherwise. Then we have the following result.

*Lemma 1:* In an optimal contract with complete information, each SU receives zero payoff by accepting the corresponding contract item. In other words, $t_k\theta_k = p_k$ for any $k \in \mathcal{K}$.

*Proof.* We prove by contradiction. Suppose that there exists an optimal contract item $(p_k, t_k)$ with $\theta_k t_k - p_k > 0$. Since PU's utility in (9) is increasing in $p_k$ and decreasing in $t_k$, the PU can increase its utility by decreasing $t_k$ until $\theta_k t_k - p_k = 0$. This contradicts with the optimality assumption, and thus completes the proof. ■

Using Lemma 1, we can replace $p_k$ by $\theta_k t_k$ for each $k \in \mathcal{K}$ and simplify the PU's utility maximization problem in (9) as

$$\max_{\{t_k \geq 0, \forall k\}} \frac{\frac{R^{dir}}{2} + \frac{1}{2}\log\left(1 + \frac{\sum_{k\in\mathcal{K}} N_k\theta_k t_k}{n_0}\right)}{1 + \sum_{k\in\mathcal{K}} N_k t_k}. \tag{10}$$

*Theorem 1:* In an optimal contract with complete information, only the contract item for the highest type is positive and all other contract items are zero. That is $(p_K, t_K) > \mathbf{0}$, and $(p_k, t_k) = \mathbf{0}$ for any $k < K$.

The proof of Theorem 1 is given in [24]. Intuitively, the highest type SUs can offer the most help to the PU with the same time allocation in Phase III.

Using Theorem 1, the optimization problem in (10) can be simplified as

$$\max_{t_K \geq 0} \frac{1}{1 + N_K t_K}\left(\frac{R^{dir}}{2} + \frac{1}{2}\log\left(1 + \frac{\theta_K N_K t_K}{n_0}\right)\right). \tag{11}$$

Since $N_K$ and $t_K$ always appear as a product in (11), we can redefine the optimization variable as $\tilde{t}_K = N_K t_K$ and rewrite (11) as

$$\max_{\tilde{t}_K > 0} \frac{1}{1 + \tilde{t}_K}\left(\frac{R^{dir}}{2} + \frac{1}{2}\log\left(1 + \frac{\theta_K \tilde{t}_K}{n_0}\right)\right). \tag{12}$$

This means the PU's optimal utility does not depend on the number of highest type users $N_K$. When $N_K$ changes, the optimal time allocation per user $t_K^*$ changes inversely proportional to $N_K$.

At this point, we have successfully simplified the PU's optimization problem from involving $2K$ variables $\{(p_k, t_k), \forall k \in \mathcal{K}\}$ in (9) to a single variable $\tilde{t}_K$ in (12).

We can show that problem (12) is a non-convex optimization problem (details in [24]). Although it is difficult to find a closed-form solution, we can use an efficient one-dimensional exhaustive search algorithm to find the global optimal solution $\tilde{t}_K^*$ [23]. We will provide numerical results in Section VII.

## IV. Feasible Contracts Under Incomplete Information

In this section, we study the necessary and sufficient conditions for a feasible contract under incomplete information. This helps us derive optimal contracts in Sections V and VI.

A feasible contract includes $K$ power-time items such that any type-$\theta_k$ SU prefers the contract item of its type, i.e., $(p_k, t_k)$, to any other contract item. Under incomplete information, a feasible contract must satisfy both the individual rationality (IR) constraint in Definition 1 and the incentive compatibility (IC) constraint defined next.

*Definition 2:* (IC: Incentive Compatibility): A contract satisfies the incentive compatibility constraint if each type-$\theta_k$ SU prefers to choose the contract item for $\theta_k$, i.e.,

$$\theta_k t_k - p_k \geq \theta_k t_j - p_j, \forall k, j \in \mathcal{K}. \tag{13}$$

In summary, the PU's optimization problem is

$$\max_{\{(p_k,t_k),\forall k\}} u_{PU}(\{(p_k, t_k), k \in \mathcal{K}\}), \tag{14}$$

$$\begin{aligned}
\text{subject to } \ & \theta_k t_k - p_k \geq \theta_k t_j - p_j, \ \forall k, j \in \mathcal{K}, \\
& \theta_k t_k - p_k \geq 0, \forall k \in \mathcal{K} \\
& t_k \geq 0, p_k \geq 0, \ \forall k \in \mathcal{K}.
\end{aligned}$$

The first two constraints correspond to IC and IR, respectively.

### A. Sufficient and Necessary Conditions for Feasibility

Next we provide several necessary and sufficient conditions for the contract feasibility.

*Proposition 1 (Necessary condition 1):* For any feasible contract $\Phi = \{(p_k, t_k), \forall k\}$, $p_i > p_j$ if and only if $t_i > t_j$.

The proof of Proposition 1 is given in [24]. Proposition 1 shows that an SU contributing more in terms of received power at the PU's receiver should receive more time allocation, and vice versa. From Proposition 1, we have the following corollary, saying that the same relay powers must have the same time allocations, and vice versa.

*Corollary 1:* For any feasible contract $\Phi = \{(p_k, t_k), \forall k\}$, $p_i = p_j$ if and only if $t_i = t_j$.

Proposition 2 shows the second necessary condition for contract feasibility.

*Proposition 2 (Necessary Condition 2):* For any feasible contract $\Phi = \{(p_k, t_k), \forall k\}$, if $\theta_i > \theta_j$, then $t_i \geq t_j$.

The proof of Proposition 2 is given in [24]. Proposition 2 shows that a higher type SU should be allocated more transmission time. Combined with Proposition 1, we know that a higher type of SU should also contribute more in terms of PU's received power.

From Propositions 1 and 2, we conclude that for a feasible contract, all power-time combination items satisfy

$$0 \leq p_1 \leq p_2 \leq ... \leq p_K, \ 0 \leq t_1 \leq t_2 \leq ... \leq t_K, \tag{15}$$

with $p_k = p_{k+1}$ if and only if $t_k = t_{k+1}$.

The previous propositions help us obtain Theorem 2 as follows.

*Theorem 2 (Sufficient and Necessary Conditions):* A contract $\Phi = \{(p_k, t_k), \forall k\}$ is feasible if and only if all the following three conditions hold:

- `Contd.a`: $0 \leq p_1 \leq p_2 \leq ... \leq p_K$ and $0 \leq t_1 \leq t_2 \leq ... \leq t_K$;
- `Contd.b`: $\theta_1 t_1 - p_1 \geq 0$;
- `Contd.c`: For any $k = 2, 3, ..., K$,

$$p_{k-1} + \theta_{k-1}(t_k - t_{k-1}) \leq p_k \leq p_{k-1} + \theta_k(t_k - t_{k-1}). \tag{16}$$

We give the proof of Theorem 2 in [24]. The conditions in Theorem 2 are essential to the optimal contract design under weakly and strongly incomplete information in Section V and Section VI.

$$\max_{\{(p_k,t_k),\forall k\}} \sum_{n_1=0}^{N} \sum_{n_2=0}^{N-n_1} \cdots \sum_{n_{K-1}=0}^{N-\sum_{i=1}^{K-2} n_i} \frac{Q_{(n_1,\dots,n_{K-1},N-\sum_{i=1}^{K-1} n_i)}}{1+\sum_{i=1}^{K-1} n_i t_i + (N-\sum_{i=1}^{K-1} n_i)t_K} \left(\frac{R^{dir}}{2} + \frac{1}{2}\log\left(1+\frac{\sum_{i=1}^{K-1} n_i p_i + (N-\sum_{i=1}^{K-1} n_i)p_K}{n_0}\right)\right). \quad (21)$$

## V. Optimal Contract Design under Weakly Incomplete Information

In this section, we will look at the weakly incomplete scenario, where the PU does not know each SU's type but knows the number of each type (i.e., $N_k$ for any $k \in \mathcal{K}$). Without loss of generality, we assume that $N_k \geq 1$ for all $k \in \mathcal{K}$. Different from the complete information case, here the PU cannot force an SU to accept certain contract item. Thus we need to consider IC constraint here (which was not an issue in the complete information case in Section III).

A conceptually straightforward approach to derive the optimal contract is to solve (14) directly. Going through this route, however, is very challenging as (14) is a non-convex and involves complicated constraints.

Here we adopt a sequential optimization approach instead. We first derive the best relay powers $\{p_k^*(\{t_k,\forall k\}),\forall k\}$ given fixed feasible time allocations $\{t_k,\forall k\}$, then derive the best time allocations $\{t_k^*,\forall k\}$ for the optimal contract, and finally show that there is no gap between $\{(p_k^*,t_k^*),\forall k\}$ and the optimal solution of (14).

*Proposition 3:* Let $\Phi = \{(p_k,t_k),\forall k\}$ be a feasible contract with fixed time allocations $\{t_k, \forall k: 0 \leq t_1 \leq \dots \leq t_K\}$. The optimal unique relay powers satisfy

$$p_1^*(\{t_k,\forall k\}) = \theta_1 t_1,$$
$$p_k^*(\{t_k,\forall k\}) = \theta_1 t_1 + \sum_{i=2}^{k} \theta_i (t_i - t_{i-1}), \forall k = 2,\dots,K. \quad (17)$$

The proof of this proposition is given in [24]. Using Proposition 3, we can simplify the PU's optimization problem in (14) as

$$\max_{\{t_k,\forall k\}} \frac{\frac{R^{dir}}{2} + \frac{1}{2}\log\left(1+\frac{\sum_{k\in\mathcal{K}} N_k(\theta_1 t_1 + \sum_{i=1}^{k}\theta_i(t_i-t_{i-1}))}{n_0}\right)}{1+\sum_{k\in\mathcal{K}} N_k t_k}, \quad (18)$$

subject to $\quad 0 \leq t_1 \leq \dots \leq t_K.$

We can further simplify (18) using Theorem 3 below.

*Theorem 3:* In an optimal contract with weakly incomplete information, only the contract item for the highest SU type is positive and all other contract items are zero. That is $(p_K,t_K) > \mathbf{0}$, and $(p_k,t_k) = \mathbf{0}$ for any $k < K$.

Theorem 3 can be proved in a similar manner as Theorem 1, and the detailed proof is given in [24].

Using Theorem 3, we can simplify the optimization problem in (18) further as

$$\max_{t_K \geq 0} \frac{1}{1+N_K t_K}\left(\frac{R^{dir}}{2} + \frac{1}{2}\log\left(1+\frac{\theta_K N_K t_K}{n_0}\right)\right). \quad (19)$$

Notice that (19) under weakly incomplete information is the same as (11) under complete information. We thus conclude that our sequential optimization approach (first over $\{p_k,\forall k\}$ and then over $\{t_k,\forall k\}$) results in no loss in optimality, as it achieves the same maximum PU's utility as in the complete information scenario.

To solve problem (19), we can use an efficient one-dimensional exhaustive search algorithm to find the global optimal solution $t_K^*$. We will provide numerical results in Section VII.

## VI. Optimal Contract Design under Strongly Incomplete Information

In this section, we study the strongly incomplete information scenario, where the PU does not know each SU's type or the number of each type (i.e., $N_k$ for each $k \in \mathcal{K}$). The PU only knows the total number of SUs $N$ and the probability $q_k$ of each SU belonging to type-$\theta_k$ (i.e., $\sum_{k\in\mathcal{K}} q_k = 1$).

Similar to the weakly incomplete information scenario, here the PU needs to consider the IC constraint since it cannot force an SU to accept a certain contract item. The difference from Section V is that here the PU does not know which type is the highest type in the system and how many SUs belong to the highest type, and thus the simple approach of only providing a positive contract item for type $\theta_K$ as in Theorem 3 may not be optimal. If the PU does that and it turns out that $N_K = 0$ in the network, then there will be no SUs participating in the cooperative communications.

The right target for the PU is design a contract to maximize the *expected* utility subject to the IC and IR constraints. As the PU knows the total number of SUs $N$, then the probability density function of the number of SUs $\{N_k,\forall k\}$ is

$$Q_{(n_1,\dots,n_{K-1},n_K=N-\sum_{i=1}^{K-1} n_i)}$$
$$:= \Pr(N_1 = n_1,\dots,N_{K-1}=n_{K-1},N_K = N-\sum_{i=1}^{K-1} N_i)$$
$$= \frac{N!}{n_1!\dots n_{K-1}!(N-\sum_{i=1}^{K-1} n_i)!} q_1^{n_1}\dots q_{K-1}^{n_{K-1}} q_K^{N-\sum_{i=1}^{K-1} n_i}. \quad (20)$$

The PU's optimization problem can be written in (21) subject to the IC ad IR constraints.

Similar to Section V, here we adopt a sequential optimization approach: we first derive the optimal relay powers $\{p_k^*(\{t_k,\forall k\}),\forall k\}$ with fixed feasible time allocations $\{t_k,\forall k\}$, and then derive the optimal time allocations $\{t_k^*,\forall k\}$ for the optimal contract. The difference is that optimality is no longer guaranteed as explained later.

*Proposition 4:* Let $\Phi = \{(p_k,t_k),\forall k\}$ be a feasible contract with fixed time allocations $\{t_k, \forall k: 0 \leq t_1 \leq \dots \leq t_K\}$, then the unique optimal relay powers satisfy

$$p_1^*(\{t_k,\forall k\}) = \theta_1 t_1,$$
$$p_k^*(\{t_k,\forall k\}) = \theta_1 t_1 + \sum_{i=2}^{k} \theta_i (t_i - t_{i-1}), \forall k = 2,\dots,K. \quad (22)$$

$$\max_{\{t_k,\forall k\}} \sum_{n_1=0}^{N} \sum_{n_2=0}^{N-n_1} \dots \sum_{n_{K-1}=0}^{N-\sum_{i=1}^{K-2} n_i} Q_{(n_1,\dots,n_{K-1},N-\sum_{i=1}^{K-1} n_i)} \frac{\left(\frac{R^{dir}}{2} + \frac{1}{2}\log\left(1+\frac{\sum_{i=1}^{K-1} n_i p_i^*(\{t_k,\forall k\})+(N-\sum_{i=1}^{K-1} n_i)p_K^*(\{t_k,\forall k\})}{n_0}\right)\right)}{1+\sum_{i=1}^{K-1} n_i t_i + (N-\sum_{i=1}^{K-1} n_i)t_K}. \quad (23)$$

subject to, $\quad 0 \le t_1 \le \dots \le t_K.$

Notice that Proposition 4 under strongly incomplete information is actually the same as Proposition 3 under weakly incomplete information. Proposition 4 can be proved in a similar manner as Proposition 3, using the fact that PU's expected utility is increasing in $\{p_k,\forall k\}$. The proof is omitted here due to space limit, and can be found in our online technical report [24]. Based on Proposition 4, we can simplify the PU's optimization problem in (21) as (23).

Note that (23) is a non-convex optimization problem and all $K$ variables are coupled in the objective function. Furthermore, the number of terms in the objective increases exponentially with the number of types $K$. Thus it is hard to solve efficiently.

Next, we propose a low computation complexity approximate algorithm, Decompose-and-Compare algorithm, to compute a close-to-optimal solution to (23) efficiently. In this algorithm, we will compare $K$ simple candidate contracts, and pick the one that yields the largest utility for the PU. There are two key steps behind this heuristic algorithm.

- *Decompose into $K$ candidate contracts:* In the $k$th candidate contract, we offer a single contract item $t_k > 0$ to SUs with a type equal to or larger than $\theta_k$, and zero for other types. The value of $t_k$ is chosen to maximize the PU's expected utility in (23).
- *Compare to achieve the balance between efficiency and uncertainty:* Among $K$ candidate contracts, we pick the one that achieves the best trade-off between efficiency and uncertainty, i.e., maximizing the PU's expected utility. For the $k$th contract, SUs with type $\theta_k$ or above will accept the contract. The larger the type, the higher positive payoff achieved by an SU. Under strongly incomplete information, it is not clear which type is the highest among all SUs in the network. Thus choosing a candidate contract with a threshold $\theta_k$ too low will leave too much payoffs to the SUs (and thus reduce the PU's expected utility), but picking the threshold too high might lead to the undesirable case that no SUs want to participate. This requires us to examine all $K$ candidate contracts and pick the one with the best performance.

In Section VII, we will use numerical results to show that the proposed Decompose-and-Compare algorithm achieves a performance very close to the optimal solution to (23) in most cases.

## VII. Numerical Results

Here we use numerical results to show the performance of the proposed contracts in different information scenarios.

### A. Complete and Weakly Incomplete Information Scenarios

As shown in Section III and Section V, the optimal contract is the same for complete and weakly incomplete information scenarios. By examining the PU's optimization in (19) that applies to both scenarios, we have the following observations.

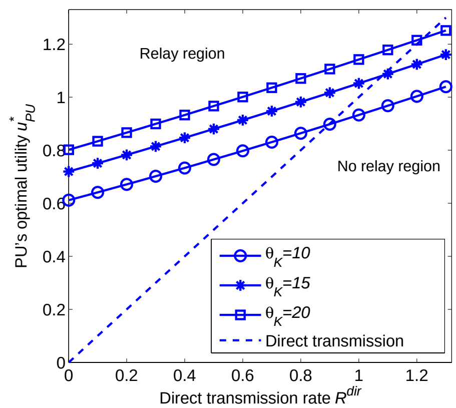


Fig. 2. PU's optimal utility $u^*_{PU}$ as a function of the PU's direct transmission rate $R^{dir}$ and the highest type $\theta_K$. The dotted baseline with 45° divides the figure into two regions: Relay Region and No Relay Region.

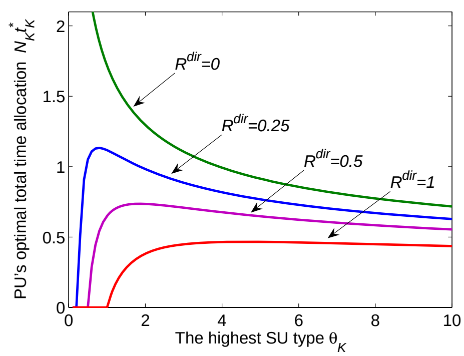


Fig. 3. PU's optimal total time allocation $N_K t^*_K$ as a function of the PU's direct transmission rate $R^{dir}$ and the highest type $\theta_K$.

*Observation 1:* Figure 2 shows that the PU's optimal utility increases in the highest SU type-$\theta_K$ and the PU's direct transmission rate $R^{dir}$.[4]

In Fig. 2, the dotted baseline denotes the rate $R^{dir}$ achieved by direct transmission only. As $R^{dir}$ increases, the PU has less incentive to share spectrum with the SUs. When $R^{dir}$ is very large, the PU chooses not to use SUs at all, which corresponds to *No Relay Region* in Fig. 2 (where the three curves are below the baseline).

For the rest of the numerical results, we will only examine the PU's choice of optimal contract, without reiterating the need to compare with $R^{dir}$ and choose direct transmission only if needed.

*Observation 2:* Figure 3 shows that the PU's optimal total

[4] Without loss of generality, we normalize $n_0$ to be 1 in the simulations.

time allocation to the highest type-$\theta_K$ SUs, $N_K t_K^*$, decreases in PU's direct transmission rate $R^{dir}$. When $R^{dir} = 0$, the PU's optimal total time allocation is strictly decreasing in $\theta_K$; when $R^{dir} > 0$, $N_K t_K^*$ first increases in $\theta_K$ and then decreases in $\theta_K$.

When direct transmission rate $R^{dir} = 0$, the PU definitely needs SU's help and will always allocate positive transmission time to the highest type-$\theta_K$ SUs. If we look at the PU's utility in (19) with $R^{dir} = 0$, the logarithmic term $\log(1 + \theta N_K t_K^*)$ plays a more important role than PU's transmission time ratio $\frac{1}{1+N_K t_K^*}$ in this case. When $\theta_K$ is small, the PU needs to allocate a large amount time to the SUs to achieve its desirable rate. When $\theta_K$ becomes large, the PU can reach a high relay rate by allocating less transmission time to the SUs. This explains why we observe a decrease of $N_K t_K^*$ in $\theta_K$. Our technical report [24] provides a rigorous proof of Observation 2 under $R^{dir} = 0$.

When direct transmission rate $R^{dir} > 0$ (the lower three curves in Fig. 3), the PU has less incentive to allocate transmission time to the SUs, especially when the highest SU type-$\theta_K$ is small. As $\theta_K$ becomes large, PU is willing to allocate more time in exchange of efficient help from SUs. As $\theta_K$ becomes very large, the PU only needs to allocate a small amount of time to the SUs in order to obtain enough relay help. The above analysis together explain why the lower three curves in Fig. 3 first increase and then decrease in $\theta_K$.

### *B. Strongly Incomplete Information Scenario*

Here we show how PU chooses the optimal contract to maximize its expected utility. As a performance benchmark, we first compute the optimal solution to the PU's expected utility maximization problem in (23) via a $K$-dimension exhaustive search. We denote the corresponding optimal solution as $E[u_{PU}]^*$. Notice that $E[u_{PU}]^*$ is often smaller than the maximum utility achieved under complete information. The performance gap is due to the strongly incomplete information. Next, we will compare the PU's expected utility achieved by the proposed Decompose-and-Compare algorithm with $E[u_{PU}]^*$.

For illustration purposes, we consider only two types of SUs: $\theta_1 < \theta_2$. The PU only knows the total number of SUs $N$ and the probabilities $q_1$ and $q_2$ of two types, with $q_1 + q_2 = 1$.

In the Decompose-and-Compare algorithm, we first consider two candidate contracts. The first candidate contract optimizes a single positive contract item $t_1 = t_2 > 0$ for both types. The second candidate contract optimizes the positive contract item $t_2 > 0$ and sets $t_1 = 0$. Then we pick the candidate contract that leads to a larger PU's expected utility as the solution of the Decompose-and-Compare algorithm.

Our numerical results (with details in [24]) show that the proposed Decompose-and-Compare algorithm performs as well as the exhaustive search when $q_1^N$ is small. In this case, the probability that at least one SU belongs to the high type-$\theta_2$ is large and the PU will focus on the high type SU(s). Even when $q_1^N$ is large, we can show that the PU's expected utility loss using Decompose-and-Compare algorithm is very small compared to the exhaustive search (e.g., less than 2% for $q_1^N = 0.81$, $\theta_1 = 4$, and $\theta_2 = 10$).

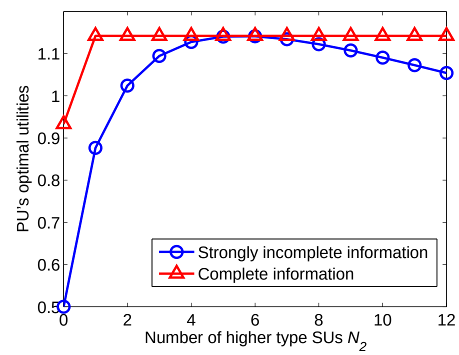


Fig. 4. Comparison among PU's optimal utilities for any SU number realization under different information scenarios. Other parameters are $q_1 = 0.5$, $N = 12$, $R^{dir} = 1$, $\theta_1 = 10$, and $\theta_2 = 20$.

Next we study how the strongly incomplete information reduces PU's utility comparing with the complete information benchmark. First, we note that PU's contract in the strongly incomplete information scenario does not depend on $N_1$ and $N_2$, as the PU targets at optimizing the expected utility and does not know the realization information. However, the actual PU's utility (not the expected value) does depend on $N_1$ and $N_2$. Figure 4 shows the PU's utility under different information scenarios and different values of $N_1$ and $N_2$ with $N_1 + N_2 = N$. The (very close to) optimal contract under strongly incomplete information can be obtained by using the Decompose-and-Compare algorithm, where $q_1^N$ approaches to zero. The optimal contract under complete information changes as $N_1$ and $N_2$ change.

*Observation 3:* Figure 4 shows that the PU's optimal utility under strongly incomplete information achieves the maximum value (close to the one under complete information) when the realized SU numbers is close to the expected value (i.e., $N_1 = N_2 = 6$ in this example as $q_1 = q_2 = 0.5$).

In Fig. 4, the largest performance gap between the two curves happens when $N_2 = 0$. In this case, the optimal contract under complete information satisfies $t_1 > 0$ and $t_2 = 0$, as there are no type-$\theta_2$ users. However, the PU under strongly incomplete information chooses $t_1 = 0$ and $t_2 > 0$ to maximize the PU's expected utility. Such mismatch means that the PU under strongly incomplete information has no SUs serving as relays, and can only achieve a utility equal to half of the direct transmission rate (only in Phase I). However, this parameter setting only happens with a very small probability $q_1^N = 0.5^{12} \approx 2.4 \times 10^{-4}$.

A more meaning comparison is the PU's *expected* utility loss due to strongly incomplete information. We can first compute the PU's expected utility under strongly incomplete information, which is the optimal objective of the PU's expected utility maximization problem in (23) (via an $K$-dimension exhaustive search). Then we can compute the PU's expected utility under complete information by calculating the weighted sum (weighted by the probability of each parameter $(N_1, N_2)$) of the 13 values on the upper curve in Fig. 4. In this example, the ratio is 0.9874. This means that the PU's expected utility loss due to strongly incomplete information is

TABLE II
A SUMMARY OF SPECTRUM TRADING LITERATURES

| Network Information | Money-Exchange | Resource-Exchange |
|---|---|---|
| **Complete** | Pricing: [3]–[6] | Stackelberg: [15]–[18] |
| **Incomplete** | Contract: [8], [9]<br>Auction: [10]–[14] | Contract: This paper<br>Bargaining: [19] |

very small (i.e., less than 1.3%).

## VIII. Related Work

Recent years have witnessed a growing body of literatures on the economic aspect of dynamic spectrum sharing. Market-driven *spectrum trading* is a promising paradigm to address the incentive issue in dynamic spectrum sharing. We can classify spectrum trading models into two types: money-exchange and resource-exchange. In the former type, SUs pay PUs in the form of (virtual) money for the usage of spectrum (e.g., [3]–[14]); in the latter type, SUs provide communication resources (e.g., the transmission power in our model) for PUs' transmissions in exchange for the usage of spectrum (e.g., [15]–[19]).

There has been extensive research on the *money-exchange* spectrum trading models, often in the form of pricing [3]–[7], contract [8], [9], and auction [10]–[14]. *Pricing* is often used when the seller knows precisely the value of the resource being sold. *Contract* is more effective in the case where the seller only knows limited information (e.g., distribution) of the buyers' evaluation for the resource. In the case where the seller has even less or no knowledge about the valuation, *Auction* becomes an effective approach by motivating the bidders bid for the resource in a truthful manner.

Money-exchange spectrum trading is most effective when PUs have some temporarily unused spectrum. However, when PUs' own demands are high or the primary channels' capacities are low (e.g., due to shadowing and deep fading), there will be hardly any resource left for sale. In this case, *resource-exchange* spectrum trading can be a better choice. *Cooperative spectrum sharing* is an effective form of resource-exchange spectrum trading [15]–[19], wherein PUs utilize SUs as cooperative relays. Such cooperation can significantly improve PUs' data rate and thus can free up spectrum resources for SUs. Existing cooperative spectrum sharing mechanisms are usually based on Stackelberg game formulations with complete information [15]–[18]. The only recent work focusing on *incomplete information* is based on a dynamic bargaining formulation between one PU and one SU [19]. In this paper, we consider the interaction between one PU and multiple SUs under incomplete information, and propose a contract-based cooperative spectrum sharing mechanism.

We summarize the key literatures of spectrum trading in Table II.

## IX. Conclusion

We study the cooperative spectrum sharing between one PU and multiple SUs, where the SUs' types are private information. We model the network as a job market, in which the PU offers the contract and each SU selects the best contract item according to its type. We study the optimal contract design for multiple information scenarios. We first provide the necessary and sufficient conditions for feasible contracts under incomplete information. For the weakly incomplete information scenario, we derive the optimal contract that achieves the same PU's utility as in the complete information benchmark. For the strongly incomplete information scenario, we propose a Decompose-and-Compare algorithm that achieves a close-to-optimal PU's expected utility. Both the PU's expected utility loss due to the suboptimal algorithm and the strongly incomplete information are small, e.g., less than 5% and 1.3% in our numerical results with two SU types.

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