Book Title: Cognitive Radio Networks

November 10, 2008

Contents

Chapter 1

Game Theory for Spectrum Sharing

Jianwei Huang and Zhu Han

Cognitive radio technology enables flexible and dynamic spectrum sharing among multiple radio networks and users and has the potential of greatly improving the spectrum utilization and network performance. This new communication paradigm, however, requires a new design and analysis framework targeting at highly flexible and distributed communication and networking. Game theory is very suitable for this task, since it is a comprehensive mathematical theory for modeling the interactions among distributed and intelligent rational decision makers. In this chapter, we discuss several game theoretical models/concepts that are highly relevant for spectrum sharing, including iterative water-filling, potential game, supermodular game, bargaining, auction, and correlated equilibrium. We also discuss several related open problems, such as the lack of proper models for dynamic and incomplete information games in this area.

1.1 Introduction

Wireless spectrum has been a tightly controlled resource worldwide since the early part of the 20th century. The traditional way of regulating the spectrum is to assign each wireless application its own slice of spectrum at a particular location. Currently, almost all spectrum licenses belong to government identities and commercial operators. Thus every new commercial service, from satellite broadcasting to wireless local-area network, has to compete for licenses with numerous existing sources, creating a state of "spectrum drought" [1]. However, recent technology advances of smart technologies in software-defined, frequency-agile, or cognitive radios [2–4], together with reforms of the government regulation policies, may enable more flexible and efficient spectrum sharing.

In cognitive radio networks, wireless devices and networks can sense, adapt, and efficiently utilize the spectrum resource to achieve the communication targets. When end-users and network operators have selfish objectives, it is natural to analyze their interactions using game theory. Even when users want to cooperate, game theory still provides a powerful mathematical framework for designing spectrum sharing algorithms with fast convergence, robust performance, and limited information exchange requirements. In this chapter, we will introduce various game theoretical models that are closely related to spectrum sharing, explain their fundamental concepts and properties, and give concrete application examples.

This chapter is organized as follows. In Section 1.2, we first review the background and trends of spectrum sharing for cognitive radios. Compared with the traditional licensed-based static spectrum allocation, the flexible and dynamic nature of spectrum sharing demands a new design and analysis framework. Game theory is a good candidate to fill in this theoretical gap. The background of game theory is introduced in Section 1.3, and several concrete models relevant to dynamic spectrum sharing are illustrated in Section 1.4. We finally conclude and discuss open problems in Section 1.5.

1.2 Spectrum Sharing

1.2.1 Current Spectrum Control Policy

There is a growing consensus that the current spectrum regulation policy is out of date. In the US, the federal government established control of the electromagnetic spectrum around

90 years ago, largely as a consequence of the communications failures associated with the sinking of the Titanic [5]. The Federal Communications Commission (FCC) was established in 1934 to be responsible for spectrum management. Since 1934 to 1990, the *command-and-control* model has been the core of the US spectrum policy. This model is based on the assumption that simultaneous transmissions within the same spectrum at the same location will cause mutual interferences and make the transmissions useless. Thus, a highly centralized control model was adopted to assign licensees to different wireless applications to maintain their service levels. For more detailed discussions on the deficiencies with current spectrum policies, see [6, 7].

There are several arguments put forward to support changing spectrum policies. First of all, there has been a rapid increase in the number of wireless users; it will be difficult to accommodate such increasing demand in the current management framework. Second, advances in communication technologies such as error control coding and digital filtering have made wireless receivers more immune to interference, which allow for the possibility of devices coexisting within the same spectrum. Third, many empirical studies have shown that the current spectrum usages are far from efficient – there are many spectrum holes (both in time and in space) that could be exploited if more flexible usage models are used. Fourth, the rapid development of cognitive radio technology, which enables the radio devices to detect the spectrum environment, find the spectrum holes, and tune the working frequency to exploit those spectrum holes, have made the dynamic spectrum sharing feasible.

1.2.2 New Spectrum Sharing Approaches

Several approaches have been taken to achieve more efficient use of the spectrum resource within the past decade. The FCC has reclaimed spectrum from the U.S. military and the TV industry and reallocate these spectrum resources to other (higher valued) wireless applications such as third-generation mobile services [1]. Another approach is applying auction mechanism in licenses allocations (e.g., [8–11]).

Two potential new spectrum assignment policies are described by the FCC Spectrum

Policy Force Report [5]: the *exclusive use* and the *commons* model. The exclusive use model urges the relaxation of the current technical and commercial limitations on the existing spectrum licenses. For example, the current licensee may still have exclusive rights to the spectrum, but could allow other secondary users to access the spectrum in several flexible ways. The transmissions of the primary and secondary users could coexist, providing that a maximum *interference temperature* constraint is not violated at the primary user' receiver(s). Or the primary user could temporarily lease the whole spectrum to secondary users when the primary service is not in operation. Several discussions on how such secondary markets could be operated can be found in [2, 12]. The commons model allows unlimited numbers of unlicensed users to share frequencies, with usage rights that are governed by technical standards or etiquettes but with no right to protection from interference. The commons model is closely related to the open spectrum access model. In either model, the FCC wants to give spectrum users maximum autonomy in the following areas: choice of uses or services that are provided on spectrum, choice of technology that is most appropriate to the spectrum environment, and the right to transfer, lease, or subdivide spectrum rights [5].

The flexible and dynamic nature of the spectrum sharing demands a new design and analysis framework. Since the secondary users typically have selfish interests and make distributed decisions, the traditional network oriented centralized optimization and control methods are no longer applicable. Game theory turns out to be an ideal analysis framework for spectrum sharing application as explained next.

1.3 Game Theory

A good mathematical theory for modeling the interactions among distributed entities in a network is game theory [13, 14], which aims at studying interactive decision problems among intelligent rational decision makers. In this section, we will briefly introduce the necessary definitions and solution concepts that are relevant to this chapter, mainly based on discussions in [15]. Other good introductions of game theory include [16–18].

1.3.1 Basic Definitions

The essential elements of a game are the *players*, the *actions*, the *payoffs* and the *information*, known collectively as the *rules of the game*.

Players are the individuals who make decisions, denoted by a set $\mathcal{M} = \{1, \cdots, M\}$. An *action* a_i is a choice player i can make. Player i's *action set* $\mathcal{A}_i$ is the set of all the choices he can make. An *action profile* $\boldsymbol{a} = \{a_i\}_{i \in \mathcal{M}}$ is a sequence of all players' actions, one from each player. For example, in an auction setting, players are the bidders and actions are the bids submitted by the bidders. A common action set for a bidder is the interval of $[0, \infty)$, i.e., he can submit any nonnegative bid. The action sets of all users other than user i is denoted as $\mathcal{A}_{-i}$.

Player i's *payoff* $s_i(\boldsymbol{a})$ is a function of the action profile $\boldsymbol{a}$ and describes how much the player gains from the game for each possible action profile. In the games we consider, a player's payoff typically equals his utility $u_i(\boldsymbol{a})$ minus his payment $c_i(\boldsymbol{a})$, i.e., $s_i(\boldsymbol{a}) = u_i(\boldsymbol{a}) - c_i(\boldsymbol{a})$.[1][2] Note that in general, we allow both a player' utility and payment to depend on the action profile. One important assumption of game theory is that all players are *rational*, i.e., they want to choose actions to maximize their payoffs.

Given players, actions, and payoffs, we can represent a game as $G = \left\{\mathcal{M}, \{\mathcal{A}_i\}_{i \in \mathcal{M}}, \{s_i\}_{i \in \mathcal{M}}\right\}$. There could be some exceptions, e.g., players' action sets are not independent. We will discuss such a case in Section 1.4.2.

A player's *information* can be characterized by an *information set*, which tells what kind of knowledge the player has at the decision instances. In order to maximize their payoffs, the players would design contingent plans known as *strategies*, which are mappings from one player's information sets to his actions. A strategy could be *pure*, in which case it only contains one deterministic action for each information set; or it could be *mixed*, in which case it specifies a set of actions each chosen according to a probability vector for each information

[1] Sometimes we also call a player' payoff the player's *surplus*. Here we assume that players' utilities are *quasilinear* with respect to *numeraire commodity*, i.e., the utilities can be measured in terms of money [19].

[2] Our notations are consistent with the conventions used in the communication literature but a little bit different from the traditional economics literature, where the utility used here is called *valuation*, and payoff used here actually refers to the *(expected) utility*. See, for example, [18].

set. A *strategy profile* is a sequence of the players' strategies, one from each player.

In a *static* games, players choose their strategies simultaneously and only once. Each player only has one information set, which is what he knows at the beginning of the game. In this case, each pure strategy corresponds to one action, and we will loosely use them interchangeably with the same notation. We will mainly focus on static games in this chapter unless otherwise stated.

A reasonable prediction of the outcome of a game is an *equilibrium*, which is a strategy profile where each player chooses a best strategy to maximize his payoff. Among several available equilibrium concepts, we focus mainly on the *Nash Equilibrium (NE)*. In a static game, an NE is a strategy profile $\boldsymbol{a}^*$ where no player can increase his payoff by deviating unilaterally, i.e.

$$s_i\left(a_i^*, a_{-i}^*\right) \geq s_i\left(a_i', a_{-i}^*\right), \forall a_i' \neq a_i^*, a_i' \in \mathcal{A}_i, a_i^* \in \mathcal{A}_i.$$

Here we use notation $\boldsymbol{a} = \left(a_i, a_{-i}\right)$, where a_{-i} represents the actions of all players except player i. One concept closely related to the NE is a player i's *best response*, which can be defined as follows

$$\mathcal{B}_i\left(a_{-i}\right) = \arg\max_{a_i \in \mathcal{A}_i} s_i\left(a_i, a_{-i}\right).$$

When $\mathcal{B}_i\left(a_{-i}\right)$ is a singleton set, it is called the *best response function* of the actions a_{-i}. It is easy to see that an NE is a fixed point of all the players' best responses, i.e., for an NE $\boldsymbol{a}^*$,

$$a_i^* \in \mathcal{B}_i\left(a_{-i}^*\right), \forall i \in \mathcal{M}.$$

Note that there may be no NE or multiple NEs in a given game.

1.3.2 Bounded Rationality and Myopic Best Response Updates

In many problems that we encounter in practice, the players only know their own payoff functions (private information assumption) at the beginning of the game. This makes it difficult for players to determine the NE since they cannot calculate the other players' best responses, and thus are not able to find an NE by solving for a fixed point. In game theory, a traditional way of dealing with this private information assumption is to assume that players know the *distributions* of other players' payoff functions and choose strategies to maximize their *expected* payoffs. This leads to the concept of a *Bayesian NE*. Most of the classical analysis of auction theory is built on this solution concept. However, assuming a *publicly known distribution assumption* is questionable for many spectrum sharing scenarios.

An alternative approach is to consider a repeated game where the players play the same static game repeatedly, and choose their actions in each round to maximize their payoffs, based on the history of the other players' actions [16]. This is effectively an incremental information revelation process, where the players' actions in each round gradually reveal their underlying payoff functions. One difficulty in this approach is that there are typically an infinite number of NEs of this new repeated game if players are able to consider both the entire history and the future of the game when making decisions.

In fact, the *intelligence* assumption, or so called *perfect rationality*, is central to classical game theory. This means that if a player knows everything that we know about the game, he can make any inferences about the situation that we can [18]. This effectively assumes that players act as a supercomputer with infinite (and free) computational capacity and can always find their best responses, no matter how complex the game is. On the other hand, in a practical game where players are people or computer agents, prefect rationality is a problematic assumption since the computation capacities are typically limited, and the time and effort of computing the best responses could be very expensive. Thus sometimes players can be better modeled with *bounded rationality* [20], especially in a complex situation such as repeated game where a rational choice would typically be based on perfect recall of the entire history and accurate forecast into the infinite future.[3]

In the context of our problems, we will consider a specific type of bounded rationality where each player does the following: in round T of the repeated game, he chooses an action $a_i^{(T)}$ according to his best response $\mathcal{B}_i\left(a_{-i}^{(T-1)}\right)$, i.e., he tries to maximize his payoff assuming that other players will choose the same actions as the immediate previous round. If every player follows the same updating rule and the action profile finally converges (i.e., everyone's

[3]The concern of limited computational capacity and bounded rationality has been the main motivation for a new research area called *computational mechanism design* [21] or *algorithmic mechanism design* [22].

action does not change from round to round), then an NE of the original static game with full information (i.e., the game where everyone knows everyone's payoff) is found. This type of update strategy is called *Myopic Best Response (MBR)* update [23, 24].

The MBR updates could be thought as a "limited memory" interpretation [20, 25] of bounded rationality, where players only remember situations in the previous round. These updates can be traced back to early studies of oligopoly. MBR update is one of the simplest learning mechanisms for the players in a game theoretic environment. In some interesting auction applications [26, 27] MBR update has been proved to be an *ex post* NE, in which there is no better strategy for a player whatever the payoffs of other agents, as long as the other players also follow MBR updates [28]. In some special game theoretic models such as *supermodular game* [29] or *potential game* [30], MBR update has very good convergence properties, though in more general settings these updates can lead to cyclic behaviors that do not converge. Interested readers are referred to [31–33] for more general discussions on learning in games.

A shortcoming of MBR update is the restricted assumption on the players' intelligence. However, we emphasize that in the cases considered here, we are often dealing with engineered system. In these cases, this assumption can simply be reviewed as a design choice. The reason for modeling this choice is that it leads to simple systems with desirable behavior.

1.4 Game Theoretical Models for Dynamic Spectrum Sharing

In this section, we will investigate several useful popular game theoretical models for dynamic spectrum sharing applications. Specifically, we study iterative water-filling, potential game, super-modular game, bargaining, auction, and correlated equilibrium in the following subsections, respectively.

1.4.1 Iterative Water-Filling

In a multi-user environment with multiple interference channels, it is important to efficiently allocate the transmission power over the different channels to maximize the transmission rate and minimize the interferences. This is a very common scenario for dynamic spectrum sharing, where multiple secondary users want to access some common open channels. In this case, a centralized approach can achieve a global optimal solution but with very high complexity and communication overhead. One of the low complexity distributed algorithms proposed in this context is iterative water-filling.

As an example, let us consider a multi-carrier system with M users and K sub-channels. The signal to interference plus noise ratio (SINR) of user i on the sub-channel k is given by

$$\gamma_i^k = \frac{p_i^k h_{ii}^k}{n_0 + \sum_{j \neq i} p_j^k h_{ji}^k}, \tag{1.1}$$

where h_{ji}^k is the channel gain from the user j's transmitter to user i's receiver at sub-channel k, p_i^k is the transmitting power of the user i at the sub-channel k, and n_0 is the thermal noise level. The rate for user i and sub-channel k (in bits/sec/Hz) is given by

$$R_i^k = \log_2(1 + \gamma_i^k). \tag{1.2}$$

Notice that here interference is treated as noise and no multi-user detection is considered. The overall rate achieved over different sub-channels for the user i is given by $R_i = \sum_{k=1}^{K} R_i^k$. User i needs to decide its transmission power vector over all sub-channels, $\boldsymbol{p} = \{p_i^k, \forall k\}$, to maximize its data rate such that the total power is no larger than $P_{\max}$.

Next, we define the Noncooperative Rate Maximization Game as

Definition 1 (Noncooperative Rate Maximization Game) *In a noncooperative rate maximization game* $G = \left[\mathcal{M}, \{\mathcal{A}_i\}_{i \in \mathcal{M}}, \{R_i\}_{i \in \mathcal{M}}\right]$*, each user* i*'s action set is*

$$\mathcal{A}_i = \{\boldsymbol{p}_i : p_i^k \geq 0, \sum_k p_i^k \leq P_{\max}\}$$

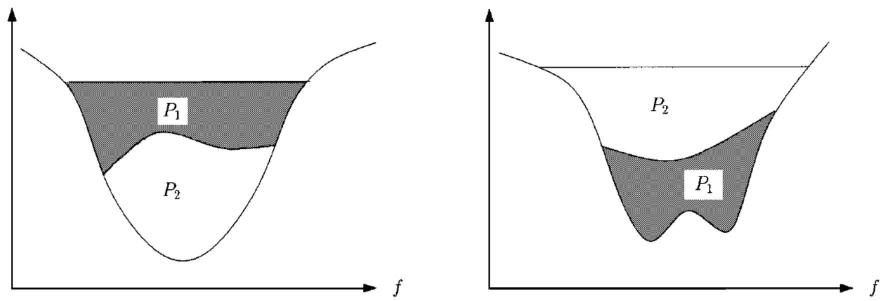

Figure 1.1: Illustration of Iterative Water-filling [57]

and the payoff is its total rate over all sub-bands.

The Nash Equilibrium of the game can be found through iterative water-filling algorithm [57]. The basic idea is to treat the interferences from the other users as the noise. Then each user employes a single-user water-filling solution iteratively based on the changes of power levels of other users. In Fig. 1.1, we illustrate a two-user example, in which the interference from the other user is treated as the noise level over different frequency. The properties for the iterative water-filling are given by the following theorems.

Theorem 1 *A Nash equilibrium always exists in the non-cooperative rate maximization game.*

Theorem 2 *Assume*

$$\max_k \frac{h_{ij}^k}{h_{ii}^k} < \frac{1}{M-1}, i \neq j, \tag{1.3}$$

then the iterative water-filling scheme globally converges to a Nash equilibrium.

Intuitively, condition (1.3) means that the interferences among users are not large enough. In particular, when the number of users M increases, the convergence condition becomes more stringent.

The iterative water-filling scheme is an efficient method for distributed resource allocation using only local information. However, when the interference is large, the convergence speed can be slow. Moreover, the Nash equilibrium is typically not optimal from system design point of view (i.e., not maximizing total rate).

To overcome the shortcoming of iterative water-filing, we can use a referee-based game to improve the performance [68, 69]. The basic idea is to introduce a referee for the non-cooperative game. The pure iterative water-filling may have multiple Nash equilibria. A referee is in charge of detecting these less efficient Nash equilibria and changing the game rule to prevent the players from falling into undesirable game outcomes. It is worth mentioning that the non-cooperative game is still played in a distributive way and the referee intervenes only when it is necessary. In [68, 69], the above idea is employed to a multicell OFDMA network to achieve an efficient distributed resource allocation.

1.4.2 Potential Game

Potential game is a class of game that has nice convergence properties to the Nash equilibria. In potential games, although players act in a noncooperative fashion, they actually implicitly work towards a common system goal characterized as *potential*. In other words, the potential serves as the mathematical bridge between non-cooperative and cooperative behaviors of the players.

We first introduce some useful definitions. More general definitions related to potential games can be found in [30].

Definition 2 (Ordinal Potential) *In a game* $G = \left[\mathcal{M}, \{\mathcal{A}_i\}_{i\in\mathcal{M}}, \{s_i\}_{i\in\mathcal{M}}\right]$*, a function* $Z : \mathcal{A}_1 \times \cdots \times \mathcal{A}_M \rightarrow R$ *is an ordinal potential for* G*, if for every* $i \in \mathcal{M}$ *and every* $a_{-i} \in \mathcal{A}_{-i}$*, we have*

$$s_i(x, a_{-i}) - s_i(y, a_{-i}) > 0 \text{ if and only if} Z(x, a_{-i}) - Z(y, a_{-i}) > 0, \forall x, y \in \mathcal{A}_i.$$

Definition 3 (Ordinal Potential Game) *A game* $G = \left[\mathcal{M}, \{\mathcal{A}_i\}_{i\in\mathcal{M}}, \{s_i\}_{i\in\mathcal{M}}\right]$ *is an ordinal potential game if it admits an ordinal potential.*

The following theorem summarizes several important properties of a potential game.

Theorem 3 *In an ordinal potential game* $G = \left[\mathcal{M}, \{\mathcal{A}_i\}_{i\in\mathcal{M}}, \{s_i\}_{i\in\mathcal{M}}\right]$,

(a) *Optimizers of the ordinal potential function* $Z(\boldsymbol{a})$ *are NEs of game* G.

(b) *If the game is finite, i.e., the number of users is finite and the strategy set* $\mathcal{A}_i$ *is finite for each user, then all improvement paths are finite and terminate at an NE.*

There are other variations of potential games, including exact, weighted, generalized ordinal, and best-response potential games. More discussions can be found in [52].

Potential games have been extensively used in studying wireless power control. For examples, the authors in [53, 54] considered the case where each player chooses a scalar power level and all players' strategy sets are decoupled. In [55], the authors generalized the results to the case of vector power choice with coupled strategy sets. Furthermore, it is pointed out in [55] that there is an interesting and general relationship existing between the NEs of potential games and the equilibria of proper autonomous dynamic systems: a potential game can be interpreted as an autonomous gradient dynamic system whose Lyapunov function is just the potential of the game. This explains the convergence results in Theorem 3.

Next we give an example based on the discussions in [55]. Consider a single-cell CDMA network with $\mathcal{M} = \{1, \cdots, M\}$ users. The received SINR of user $i \in \mathcal{M}$ is

$$\gamma_i(\boldsymbol{p}) = \frac{p_i h_i}{n_0 + \sum_{j\neq i} p_j h_j},$$

where h_i is the channel gain from user i's transmitter to the base station. Each user $i \in \mathcal{M}$ wants to solve the following power minimization problem

$$\begin{aligned} \text{minimize} \quad & p_i \\ \text{subject to} \quad & f_i(\gamma_i(\boldsymbol{p})) \geq \gamma_i^{thresh}, \\ \text{variables} \quad & p_i \in [0, P_i^{\max}]. \end{aligned} \tag{1.4}$$

Here $P_i^{\max}$ is the maximum power, f_i is the QoS function, and γ_i^{thresh} is the QoS threshold. It is shown in [55] that solving Problem (1.4) for all users is equivalent of finding the NE of a non-cooperative game $G = \left[\mathcal{M}, \mathcal{A}, \{\log(P_i^{\max} - p_i)\}_{i\in\mathcal{M}}\right]$, with the *coupled* action set is

$$\mathcal{A} = \left\{\boldsymbol{p} : f_i(\gamma_i(\boldsymbol{p})) \geq \gamma_i^{thresh}, p_i \in [0, P_i^{\max}], \forall i \in \mathcal{M}\right\}.$$

Furthermore, the game G admits a potential function

$$Z(\boldsymbol{p}) = \sum_{i\in\mathcal{M}} \log(P_i^{\max} - p_i).$$

We can then maximize function $Z(\boldsymbol{p})$ over set $\mathcal{A}$, and the corresponding maximizer(s) will be the NE(s) of game G, and thus the optimal solution(s) of Problem (1.4) for all users.

1.4.3 Supermodular Game

Supermodular game has many practical applications in economics. A key feature of supermodular game is the "strategic complementarities" – if a player chooses a higher action, the others want to do the same thing. Supermodular game has nice properties in terms of the existence and achievability of NE. We first introduce some useful definitions, whereas more general discussions can be found in [29].

Definition 4 (Sublattice) *A real i-dimensional set $\mathcal{V}$ is a sublattice of $\Re^i$ if for any two elements $a, b \in \mathcal{V}$, the component-wise minimum (i.e., $a \wedge b$) and the component-wise maximum (i.e., $a \vee b$) are also in $\mathcal{V}$. In particular, a compact sublattice has a (component-wise) smallest and largest element. Any compact (one-dimensional) interval is a sublattice of $\mathbb{R}$.*

Definition 5 (Function with increasing differences) *A twice differentiable function f has increasing differences in variables (x, t) if $\partial^2 f/\partial x \partial t \geq 0$ for any feasible x and t.*[4]

Definition 6 (Supermodular function) *A function f is supermodular in $\boldsymbol{x} = (x_1, .., x_i)$ if it has increasing differences in (x_i, x_j) for all $i \neq j$.*

[4] If we choose x to maximize a twice differentiable function $f(x,t)$, then the first order condition gives $\partial f(x,t)/\partial x|_{x=x^*} = 0$, and the optimal value x^* increases with t if $\partial^2 f/\partial x \partial t > 0$.

Definition 7 (Supermodular game) *A game* $G = \left[\mathcal{M}, \{\mathcal{A}_i\}_{i\in\mathcal{M}}, \{s_i\}_{i\in\mathcal{M}}\right]$ *is supermodular if for each player* $i \in \rangle$, (a) *the strategy space* $\mathcal{P}_i$ *is a nonempty and compact sublattice, and* (b) *the payoff function* s_i *is continuous in all players' strategies, is supermodular in player* i*'s own strategy, and has increasing differences between any component of player* i*'s strategy and any component of any other player's strategy.*

The following theorem summarizes several important properties of a supermodular game.

Theorem 4 *In a supermodular game* $G = \left[\mathcal{M}, \{\mathcal{A}_i\}_{i\in\mathcal{M}}, \{s_i\}_{i\in\mathcal{M}}\right]$,

(a) *The set of NEs is a nonempty and compact sublattice and so there is a component-wise smallest and largest NE.*

(b) *If the users' best responses are single-valued, and each user uses MBR updates starting from the smallest (largest) element of its strategy space, then the strategies monotonically converge to the smallest (largest) NE.*

(c) *If each user starts from any feasible strategy and uses MBR updates, the strategies will eventually lie in the set bounded component-wise by the smallest and largest NE. If the NE is unique, the MBR updates globally converge to that NE from any initial strategies.*

In wireless communications, supermodular game has been used to design various power control algorithms, e.g, [34–36]. Next we give an example based on [36], showing how supermodular game theory can help to analyze the properties of a power control algorithm.

Consider an ad hoc network with a set $\mathcal{M} = \{1, ..., M\}$ of distinct node pairs. As shown in Fig. 1.2, each pair consists of one dedicated transmitter and one dedicated receiver[5]. The motivating example for this model is multiple secondary users sharing the same common open channel in a distributed fashion.

The channel gain between user i's transmitter and user j's receiver is denoted by h_{ij}. Note that in general $h_{ij} \neq h_{ji}$, since the latter represents the gain between user j's transmitter

[5]For example, this could represent a particular schedule of transmissions determined by a routing and MAC protocol.

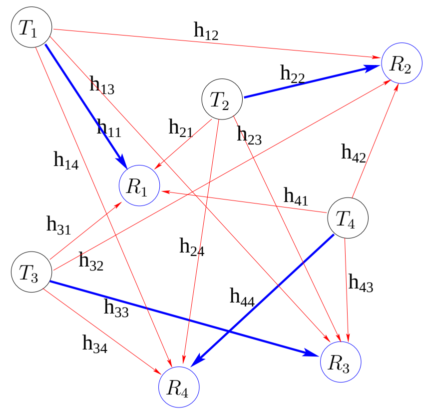

Figure 1.2: An example wireless network with four users (pairs of nodes) (T_i and R_i denote the transmitter and receiver of "user" i, respectively).

and user i's receiver. Each user i's quality of service is characterized by a utility function $u_i(\gamma_i)$, which is an increasing and strictly concave function of the received SINR,

$$\gamma_i(\boldsymbol{p}) = \frac{p_i h_{ii}}{n_0 + \sum_{j \neq i} p_j h_{ji}}, \tag{1.5}$$

where $\boldsymbol{p} = (p_1, \cdots, p_i)$ is a vector of the users' transmission powers and n_0 is the background noise power. The users' utility functions are coupled due to mutual interference. An example utility function is a *logarithmic utility function* $u_i(\gamma_i) = \theta_i \log(\gamma_i)$, where θ_i is a user dependent priority parameter (e.g., related to the long term achievable rate or queue length [76]).[6]

The problem we consider is to specify $\boldsymbol{p}$ to maximize the utility summed over all users, where each user i must satisfy a transmission power constraint $p_i \in \mathcal{P}_i = \left[P_i^{\min}, P_i^{\max}\right]$,

$$\max_{\{\boldsymbol{p}: p_i \in \mathcal{P}_i \, \forall i\}} \sum_{i=1}^{i} u_i(\gamma_i(\boldsymbol{p})). \tag{P1}$$

Note that a special case is $P_i^{\min} = 0$; i.e., the user may choose not to transmit.

We propose an Asynchronous Distributed Pricing (ADP) algorithm to solve Problem P1. We first describe the algorithm, and then show how we can interpret the algorithm

[6] In the high SINR regime, logarithmic utility approximates the Shannon capacity $\log(1+\gamma_i)$ weighted by θ_i. For low SINR, a user's rate is approximately linear in SINR, and so this utility is proportional to the logarithm of the rate.

as a *fictitious* supermodular game. This enables us to easily characterize the convergence behavior of the algorithm.

In the ADP algorithm, each user announces a single price π_i and sets its transmission power p_i based on the prices announced by other users. Prices and powers are asynchronously updated. For $i \in \mathcal{M}$, let $T_{i,p}$ and $T_{i,\pi}$, be two unbounded sets of positive time instances at which user i updates its power and price, respectively. The complete algorithm is given in Algorithm 1. Note that in addition to being asynchronous across users, each user also need not update its power and price at the same time.

Algorithm 1: The ADP Algorithm

(1) INITIALIZATION: For each user $i \in \mathcal{M}$ choose some power $p_i(0) \in \mathcal{P}_i$ and price $\pi_i(0) \geq 0$.

(2) PRICE UPDATE: At each $t \in T_{i,\pi}$, user i updates its price according to

$$\pi_i(t) = -\frac{\partial u_i\left(\gamma_i\left(\boldsymbol{p}(t)\right)\right)}{\partial\left(\sum_{j\neq i} p_j(t) h_{ji}\right)}.$$

(3) POWER UPDATE: At each $t \in T_{i,p}$, user i updates its power according to

$$p_i(t) = \arg\max_{\hat{p}_i \in \mathcal{P}_i}\left(u_i\left(\gamma_i\left(\hat{p}_i, p_{-i}(t)\right)\right) - \hat{p}_i \sum_{j\neq i} \pi_j(t) h_{ij}\right).$$

We next characterize the convergence of the ADP algorithm by considering the following *Fictitious Power-Price (FPP) control game*,

$$G_{FPP} = [\mathcal{FW} \cup \mathcal{FC}, \{\mathcal{P}_i^{\mathcal{FW}}, \mathcal{P}_i^{\mathcal{FC}}\}_{i\in\mathcal{M}}, \{s_i^{\mathcal{FW}}, s_i^{\mathcal{FC}}\}_{i\in\mathcal{M}}].$$

Here the players are from the union of the sets $\mathcal{FW}$ and $\mathcal{FC}$, which are both copies of $\mathcal{M}$. $\mathcal{FW}$ is a *fictitious power player set*; each player $i \in \mathcal{FW}$ chooses a power p_i from the strategy set $\mathcal{P}_i^{\mathcal{FW}} = \mathcal{P}_i$ and receives payoff

$$s_i^{\mathcal{FW}}\left(p_i; p_{-i}, \pi_{-i}\right) = u_i\left(\gamma_i\left(\boldsymbol{p}\right)\right) - \sum_{j\neq i} \pi_j h_{ij} p_i. \tag{1.6}$$

$\mathcal{FC}$ is a *fictitious price player set*; each player $i \in \mathcal{FC}$ chooses a price π_i from the strategy set $\mathcal{P}_i^{\mathcal{FC}} = [0, \bar{\pi}_i]$ and receives payoff

$$s_i^{\mathcal{FC}}(\pi_i; \boldsymbol{p}) = -\left(\pi_i + \frac{\partial u_i(\gamma_i(\boldsymbol{p}))}{\partial\left(\sum_{j\neq i} p_j h_{ji}\right)}\right)^2. \quad (1.7)$$

Here $\bar{\pi}_i = \sup_{\boldsymbol{p}} |\frac{\partial u_i(\gamma_i(\boldsymbol{p}))}{\partial\left(\sum_{j\neq i} p_j h_{ji}\right)}|$, which could be infinite for some utility functions.

In game G_{FPP}, each user in the ad hoc network is split into two fictitious players, one in $\mathcal{FW}$ who controls power p_i and the other one in $\mathcal{FC}$ who controls price π_i. Although users in the real network cooperate with each other by exchanging interference information (instead of choosing prices to maximize their surplus), each fictitious player in G_{FPP} is selfish and maximizes its own payoff function.

Denote $CR_i(\gamma_i) = -\frac{\gamma_i u_i''(\gamma_i)}{u_i'(\gamma_i)}$, and let $\gamma_i^{\min} = \min\{\gamma_i(\boldsymbol{p}) : p_i \in \mathcal{P}_i \, \forall i\}$ and $\gamma_i^{\max} = \max\{\gamma_i(\boldsymbol{p}) : p_i \in \mathcal{P}_i \, \forall i\}$. If for each user $i \in \mathcal{M}$, $P_i^{\min} > 0$ and $CR_i(\gamma_i) \in [1, 2]$ for all $\gamma_i \in [\gamma_i^{\min}, \gamma_i^{\max}]$, then we can show that G_{FPP} is a supermodular game, which means that Problem P1 has a unique optimal solution to which the ADP algorithm globally converges.

1.4.4 Bargaining

Bargaining games refer to situations where two or more players must reach agreement regarding how to distribute a good or monetary amount. Each player prefers to reach an agreement in these games rather than abstain from doing so; however, each prefers the agreement that maximize his own interests. Bargaining can be analyzed using the cooperative game theory as follows [58–60]:

Definition 8 (M-person bargaining problem) *Let $\mathcal{M} = \{1, 2, \ldots, M\}$ be the set of players. Let $\mathbf{S}$ be a closed and convex subset of $\Re^M$ to represent the set of feasible payoff allocations that the players can get if they all work together. Let u_{min}^i be the minimal payoff that the i^{th} player will expect; otherwise, he/she will not cooperate. Suppose $\{u_i \in \mathbf{S} | u_i \geq u_{min}^i, \forall i \in \mathcal{M}\}$ is a nonempty bounded set. Define $\mathbf{u}_{min} = (u_{min}^1, \ldots, u_{min}^M)$,* *then the pair* $(\mathbf{S}, \mathbf{u}_{min})$ *is called a* M*-person bargaining problem.*

Within the feasible set $\mathbf{S}$, we define the notion of Pareto optimal as a selection criterion for the bargaining solutions.

Definition 9 (Pareto optimality) *An allocation* $\boldsymbol{u} = (u_1, \ldots u_M)$ *is Pareto optimal if and only if there does not exist an allocation* $\boldsymbol{u}' = (u'_1, \ldots u'_M)$*, such that* $u'_i \geq u_i$ *for all* i *and* $u'_j > u_j$ *for at least one* j*. In other words, there exists no other allocation that increases the performance for some users without decreases performance for some other users.*

Next we discuss some possible bargaining solutions. In general, there might be an infinite number of Pareto optimal points, and we need further criteria to select a bargaining result. A possible criterion is the fairness. One commonly used fairness criterion for wireless resource allocation is max-min [61], where the performance of the user with the worst channel conditions is maximized. This criterion penalizes the users with good channels and as a result generates inferior overall system performance. Another more reasonable solution concept is the Nash Bargaining Solution (NBS) [60]. In an NBS, after the minimal requirements are satisfied for all users, the rest of the resources are allocated proportionally to users according to their conditions. As a result, NBS provides a unique and fair Pareto optimal operation point under the following conditions.

Definition 10 (Nash Bargaining Solution) $\bar{\mathbf{u}}$ *is said to be a Nash Bargaining Solution in* $\mathbf{S}$ *for* $\mathbf{u}_{min}$*, i.e.,* $\bar{\mathbf{u}} = \phi(\mathbf{S}, \mathbf{u}_{min})$*, if the following Axioms are satisfied:*

1. *Individual Rationality:* $\bar{u}_i \geq u^i_{min}, \forall i$.
2. *Feasibility:* $\bar{\mathbf{u}} \in \mathbf{S}$.
3. *Pareto Optimality: For every* $\hat{\mathbf{u}} \in \mathbf{S}$*, if* $u_i \geq \bar{u}_i, \forall i$*, then* $\hat{u}_i = \bar{u}_i, \forall i$.
4. *Independence of Irrelevant Alternatives: If* $\bar{\mathbf{u}} \in \mathbf{S}' \subset \mathbf{S}$*,* $\bar{\mathbf{u}} = \phi(\mathbf{S}, \mathbf{u}_{min})$*, then* $\bar{\mathbf{u}} = \phi(\mathbf{S}', \mathbf{u}_{min})$.

5. *Independence of Linear Transformations: For any linear scale transformation* ψ, $\psi(\phi(\mathbf{S}, \mathbf{u}_{min})) = \phi(\psi(\mathbf{S}), \psi(\mathbf{u}_{min}))$.

6. *Symmetry: If* $\mathbf{S}$ *is invariant under all exchanges of agents,* $\phi_j(\mathbf{S}, \mathbf{u}_{min}) = \phi_{j'}(\mathbf{S}, \mathbf{u}_{min})$, $\forall j, j'$.

Axioms 4-6 are called axioms of fairness. The irrelevant alternative axiom asserts that eliminating the feasible solutions that would not have been chosen shall not affect the NBS solution. Axiom 5 asserts that the bargaining solution is scale invariant. Symmetry axiom asserts that if the feasible ranges for all users are completely symmetric, then all users have the same solution.

The NBS satisfying the above axioms can be obtained by the following optimization [60].

Theorem 5 (Existence of NBS) *There is a solution function* $\phi(\mathbf{S}, \mathbf{u}_{min})$ *that satisfies all six axioms in Definition 10 and can be computed as*

$$\phi(\mathbf{S}, \mathbf{u}_{min}) \in \arg \max_{\bar{\mathbf{u}} \in \mathbf{S}, \bar{u}_i \geq u^i_{min}, \forall i} \prod_{i=1}^{M} \left(\bar{u}_i - u^i_{min}\right). \tag{1.8}$$

Two other bargaining solutions have been proposed as alternatives to the NBS are the Kalai-Smorodinsky Solution (KSS) and the Egalitarian Solution (ES). Details can be found in [62].

There are many applications in the wireless networks using bargaining solutions, e.g., OFDMA Resource Allocation [63], ad hoc Network [65], Mesh Networks [66], and Multimedia Transmission [67]. In [64], the authors considered using bargaining to achieve efficient dynamic spectrum sharing. The authors defined a general framework for the spectrum access problem based on several definitions of system utilities. By reducing the spectrum allocation problem to a variant of the graph coloring problem, the global optimization problem is shown to be NP-hard. A general approximation methodology is provided through vertex labeling. The paper investigated two strategies: a centralized strategy where a central server calculates an allocation assignment based on global knowledge, and a distributed approach where devices collaborate to bargain over local channel assignments towards global optimization. The experimental results show that the bargaining based allocation algorithms can dramatically reduce interferences and lead to an order of magnitude throughput improvement compared with a naive approach.

1.4.5 Auction

Auctions are suitable to model markets where the seller(s) and buyer(s) have asymmetric information. For example, consider an exclusive use spectrum sharing model, where the spectrum broker (i.e., the representative of the spectrum owner or licensee) has a piece of spectrum for sale in a secondary market. However, the broker himself may not have an accurate estimate of the secondary users' value of the spectrum since the utility functions of secondary users are typically private information. One way for the broker to extract information from the secondary users is through an auction process.

The theory of auctioning indivisible goods (single-unit or multiple-unit) has been relatively well developed [38], but the related results cannot be directly applied to spectrum sharing where the resource should typically allocated to more than one user (precluding single-unit indivisible auctions) and it is often difficult to divide the resources into well-defined bundles (precluding multi-unit indivisible auction). Next we will focus our discussions on the *share auction*, or *divisible auction*.

Share Auction

A share auction [39–41] is concerned with allocating a perfect divisible good among a set of bidders. The most commonly used example in the literature comes from the financial market (such as the auction of treasury notes) [42–44]. There are two basic pricing structures in a share auction. In a uniform-price auction, all the winners (typically more than one) get some portions of the good and pay the same unit price. In a discriminatory pricing auction (sometimes called pay-you-bid auction [40]), winning bids are filled at the bid price. Much of the results mentioned above focus on examining how different pricing and information structures affect the auction results such as the final price, seller's revenue and the allocations of the divisible good.

Compared with the well-studied single-unit good auction, where bidders typically submit one dimensional bid in the auction, some share auctions allow bidders to submit multiple combinations of price and quantity as the bids (e.g. [42, 43]. This significantly complicates the auction design since the bidders have large strategy spaces. When using a share auction to allocate resources such as bandwidth in communication networks, researchers typically adopt simple one-dimensional bidding rules as in [46–51]. In these cases, the allocations to the users are proportional to the bids.

Let us consider how share auction can be used in spectrum sharing. Consider a wireless network model that is similar as the one described in Section 1.4.3. The definitions on users (node pairs), channel gains, and utility functions are the same. The key difference here is that we do not assume that each user has a separate transmission constraint. Instead, we consider the case where there is a measurement point in the network. The aggregated interference generated by all users at the measurement point should be no larger than a threshold P, i.e.,

$$\sum_{i=1}^{i} p_i h_{i0} \leq P. \tag{1.9}$$

Here h_{i0} is the channel gain from user i's transmitter to the measurement point. The system model is shown in Fig. 1.3.

We consider two simple one-dimensional share auction mechanisms (SINR- and power-based). In both share auctions, users submit one-dimensional bids representing their willingness to pay, and the manager simply allocates the received power in proportion to the bids. The users then pay an amount proportional to their SINR (or power). The manager announces a nonnegative reserve bid β to ensure the uniqueness of the auction result. We assume that it is a complete information game, i.e., all users' utilities and all channel gains are known to all users.

Share Auction Mechanisms:

1. The manager announces a reserve bid $\beta \geq 0$, and a price $\pi^s > 0$ (in an SINR auction)

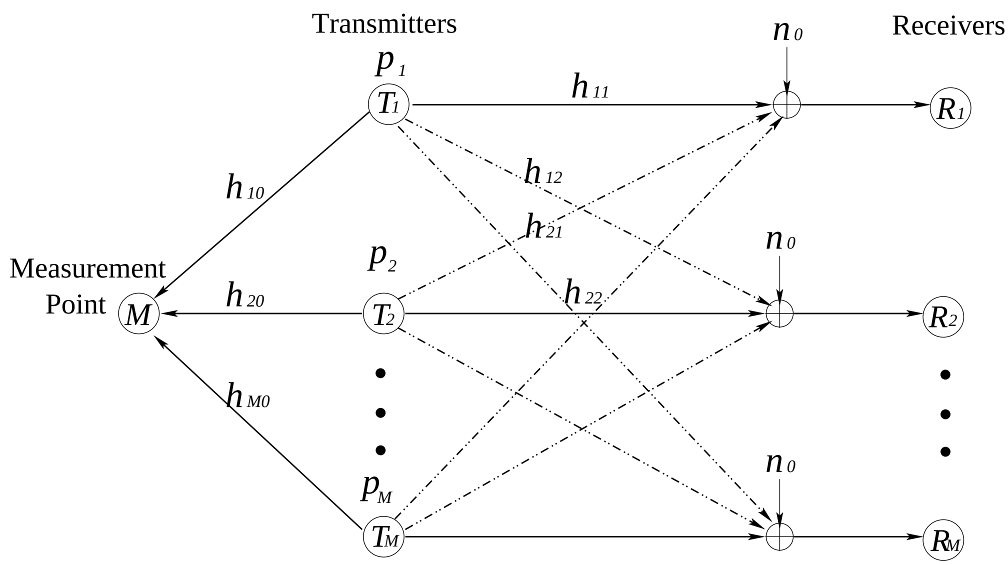

Figure 1.3: Spectrum Sharing with One Measurement Point

or $\pi^p > 0$ (in a power auction).

2. After observing β, π^s (or π^p), user $i \in \{1, ..., i\}$ submits a bid $b_i \geq 0$.

3. The manager keeps reserve power p_0, and allocates to each user i a transmission power p_i so that the received power at the measurement point is proportional to the bids, i.e.,

$$p_i h_{i0} = \frac{b_i}{\sum_{j=1}^{i} b_i + \beta} P, \text{ and } p_0 = \frac{\beta}{\sum_{j=1}^{i} b_i + \beta} P. \tag{1.10}$$

The resulting SINR for user i is

$$\gamma_i(\boldsymbol{p}) = \frac{p_i h_{ii}}{n_0 + \sum_{j \neq i} p_j h_{ji} + p_0 h_{0i}}, \tag{1.11}$$

where h_{0i} is the channel gain from the manager (measurement point) to user i's receiver[7]. If $\sum_{i=1}^{i} b_i + \beta = 0$, then $p_i = 0$.

4. In an SINR (power) auction, user i pays $C_i = \pi^s \gamma_i$ ($C_i = \pi^p p_i h_{i0}$).

These auction mechanisms differ from some previously proposed auction-based network

[7] If $h_{0i} = 0$ for all $i \in \{1, ..., M\}$, then the manager does not interfere with the users and many of the results in the following section still hold. However, in the co-located case, we have $h_{0i} = 1$ for all i.

resource allocation schemes (e.g., [46, 49]) in that the bids here are not the same as the payments. Instead, the bids are signals of willingness to pay. The manager can therefore influence the NE by choosing β and π^s (or π^p). This alleviates the typical inefficiency of the NE, and in some cases allows us to achieve socially optimal solutions.

It can be shown that under properly chosen price π^s (π^p, respectively), the SINR auction (power auction, respectively) can achieve fair (efficient, respectively) allocation. In a fair allocation, users achieve the same SINR if they have the same utility function regardless of the network topology and channel conditions. In an efficient allocation, the total utility of the network is maximized.

The above analysis assumes that all users' utilities and all channel gains are known to all users (i.e., complete information game). As a result, the NE can be calculated in one shot (with fixed price). In practice, however, users may only know limited local information. In that case, it can be shown that users can still achieve the NE in a distributed fashion by following best response dynamics.

1.4.6 Correlated Equilibrium

In this subsection, we investigate a special kind of equilibrium, correlate equilibrium. In 2006, Nobel Prize was awarded to Robert J. Aumann for his contribution to propose the concept of correlated equilibrium [70, 71]. Unlike Nash equilibrium in which each user only considers its own strategy, correlated equilibrium achieves better performance by allowing each user to consider the joint distribution of users' actions. In other words, each user needs to consider the others' behaviors to see if there are mutual benefits to explore. It has been shown that the correlated equilibrium can be better than the convex hull of the Nash equilibria [70, 71].

If a user follows a single action in every possible attainable situation (i.e., information set) in a game, the action is called pure strategy. In the case of mixed strategies, the user will follow a probability distribution over different possible actions. In Table 1.1, we illustrate an example of two secondary users (row player or column player) with different actions 0 or 1. The payoffs for both users are shown in the parenthesis. In Table 1.1 (a), we list the payoffs for two users taking an action of 0 and 1. 0 means transmitting less aggressively, while 1 means transmitting more aggresively. We can see that when two users both choose action 0, they have the best overall payoff. But if one user transmits more aggressively using action 1 while the other still plays action 0, the aggressive user achieves a better payoff while the other user has a lower payoff, and the overall benefit is reduced. However, if both users transmit aggressively using action 1, both users obtain the lowest payoff. In Table 1.1 (b), we show two Nash equilibria, where one of the user dominates the other. The dominating user has the payoff of 6 and the dominated user has the payoff of 3, which is unfair. In Table 1.1 (c), we show the mixed Nash equilibrium where two users have the probability 0.75 for action 0 and 0.25 for action 1, respectively. The payoff for each user is 4.5.

Table 1.1: Two secondary users game (from left to right): (a) payoff table; (b) Nash Equilibrium; (c) Mixed Nash Equilibrium; (d) Correlated Equilibrium.

	0	1
0	(5,5)	(6,3)
1	(3,6)	(0,0)

	0	1
0	0	(0 or 1)
1	(1 or 0)	0

	0	1
0	9/16	3/16
1	3/16	1/16

	0	1
0	0.6	0.2
1	0.2	0

In the case of correlated equilibrium, a strategy profile is chosen randomly according to a certain distribution. Given the recommended strategy, it is to the players' best interests to conform to this strategy. In Table 1.1 (b) and (c), the Nash equilibria and mixed Nash equilibria are all within the set of correlated equilibria. In Table 1.1 (d), we show an example where the correlated equilibrium is outside the convex hull of the Nash equilibrium. Notice that the joint distribution is not the product of two users' probability distributions, i.e., two users' actions are not independent. Moreover, the payoff for each user is 4.8 which is higher than that of the mixed strategy.

We define the correlated equilibrium in a formal way.

Definition 11 (Correlated Equilibrium) *A probability distribution p is a correlated equilibrium of game* $G = \left[\mathcal{M}, \{\mathcal{A}_i\}_{i\in\mathcal{M}}, \{R_i\}_{i\in\mathcal{M}}\right]$*, if and only if, for all* $i \in \mathcal{M}$*,* $a_i \in \mathcal{A}_i$*, and*

$a_{-i} \in \mathcal{A}_{-i}$,

$$\sum_{a_{-i} \in \mathcal{A}_{-i}} p(a_i, a_{-i})[s_i(a_i', a_{-i}) - s_i(a_i, a_{-i})] \leq 0, \forall a_i' \in \mathcal{A}_i. \tag{1.12}$$

By dividing inequality in (1.12) with $p(a_i) = \sum_{a_{-i} \in \mathcal{A}_{-i}} p(a_i, a_{-i})$*, we have*

$$\sum_{a_{-i} \in \mathcal{A}_{-i}} p(a_{-i}|a_i)[u_i(a_i', a_{-i}) - s_i(a_i, a_{-i})] \leq 0, \forall a_i' \in \mathcal{A}_i. \tag{1.13}$$

The inequality (1.13) means that when the recommendation to user i is to choose action a_i, then choosing action a_i' instead of a_i cannot obtain a higher expected payoff to i.

We note that the set of correlated equilibria is nonempty, closed and convex in every finite game. Moreover, it may include the distribution that is not in the convex hull of the Nash equilibrium distributions. In fact, every Nash equilibrium is a correlated equilibrium and Nash equilibria correspond to the special case where $p(a_i, a_{-i})$ is a product of each individual user's probability for different actions, i.e., the strategies of the different players are independent [70–72]. The correlated equilibrium can be calculated via linear programming. If only local information is available, some learning algorithms such as non-regret learning can achieve the correlated equilibrium with probability 1 [72].

Using the correlated equilibrium concept, there are many possible applications for cognitive radios such as power control and spectrum access [73–75]. For example, the distributive users adjust their transmission probabilities over the available channels, so that the collisions are avoided and the users' benefits are optimized. As a result, the spectrum utilization efficiency and fairness among the distributive users can be improved. To learn the correlated equilibrium in a distributed manner, the adaptive no-regret algorithm is proposed in [73] using past history. The proposed learning algorithm converges to a set of correlated equilibria with probability one. Learning schemes can achieve better equilibria using only the past history and without requiring more signaling and overhead. The complexity of learning algorithms can be relatively high. Moreover, there is a tradeoff between convergence speed and complexity. To achieve fast convergence speed, the complexity of the learning algorithms can be high. Some simple learning algorithms have been proven to converge to the optimal solution with sufficiently long learning time. However, long learning time causes a problem in a network with high mobility, where network topologies and channel conditions have changed before the learning converges. Moreover, if the non-cooperative competition is severe, the learning algorithms might converge slowly, fluctuate, or become very sensitive to randomness. So the learning schemes based on correlated equilibrium can only achieve good performance in the situations where the non-cooperative competition is not severe, there is an achievable gap between the Nash equilibria and the optimums, and the network mobility is sufficiently low.

1.5 Conclusions and Open Problems

Cognitive radio is a revolutionary wireless communication paradigm that can achieve much higher spectrum efficiency than the existing systems. Many technical challenges, however, still remain to be solved to make this vision a reality. In particular, the distributed and dynamic nature of spectrum sharing requires a new design and analysis framework. Game theory provides a nature solution for this challenging task. In this book chapter, we describe several game theoretical models that have been successfully used to solve various spectrum sharing problems.

We want to mention that most discussed models rely on the concept of Nash equilibrium in a static game with complete information. Although mathematically convenient, this may not be the most suitable game theoretical model in practice. For example, instead of participating the game only once as in a static game, secondary and primary users might interact repeatedly within a reasonable long time frame. In this case, the users make decisions not only based on the current network conditions but also on the past interaction history. A model of repeated games (either finite or infinite) will be more suitable. Moreover, the complete information assumption is difficult to be satisfied in practice, due the fast changing nature of the wireless channels and the bandwidth requirement to exchange channel measurements. Without complete channel information, the best users can do is to maximize their expected payoff based on their own beliefs of the unknown network information. How these beliefs are initialized and updated will affect the results of the game. Some preliminary work has been reported along these directions [77, 78] and definitely more are needed.

References

[1] G. Staple and K. Werbach, "The end of spectrum scarcity," *IEEE Spectrum*, pp. 48–52, March 2004.

[2] "Order and further notice of proposed rule making on smart radios," Federal Communications Commission report 03-322, 2003.

[3] "FCC: workshop on cognitive radio technologies," http://www.fcc.gov/oet/cognitiveradio/, May 2003.

[4] J. Mitola III, "Cognitive radio for flexible mobile multimedia communications," in *Prof. of IEEE International Workshop on Mobile Multimedia Communications*, pp. 3-10, San Diego, CA, 1999.

[5] "Spectrum policy task force report," Federal Communications Commission, US, Nov. 2002.

[6] T. W. Hazlett, "The wireless craze, the unlimited bandwidth myth, the spectrum auction faux pas, and the punchline to ronald coase's 'big joke': an essay on airwave allocation policy." *AEI-Brookings Joint Center for Regulatory Studies Working Paper 01-02*, January 2001.

[7] R. P. Margie, "Efficiency, predictability, and the need for an improved interference standard at the FCC," *Telecommunications Policy Research Conference*, 2003.

[8] "Auctions home page," Federal communications commission, http://www.fcc.gov/wtb/auctions.html.

[9] P. Klemperer, "How (not) to run auctions: The european 3G telecom auctions," *European Economic Review*, vol. 46, no. 4-5, pp. 829-845, May 2002.

[10] K. Binmore and P. Klemperer, "The biggest auction ever: The sale of the British 3G telecom licenses," *The Economic Journal*, vol. 112, March 2002.

[11] J. Cable, A. Henley, and K. Holland, "Pot of gold or winner's curse? An event study of the auctions of 3G mobile telephone licences in the UK," *Fiscal Studies*, vol. 23, no. 4, pp. 447–462, 2002.

[12] "The development of secondary markets - report and order and further notice of proposed rulemaking," Federal Communications Commission Report 03-113, 2003.

[13] J. von Neumann and O. Morgenstern, *Theory of games and economic behavior*. Princeton Univeristy Press, 1947.

[14] J. Nash, "Equilibrium points in n-person games," in *Proceedings of the National Academy of Sciences*, vol. 36, pp. 48–49, 1950.

[15] E. Rasmusen, *Games and information: an introduction to game theory*, 3rd ed. Blackwell Publishers, 2001.

[16] D. Fudenberg and J. Tirole, *Game theory*. MIT Press, 1991.

[17] R. Gibbons, *Game theory for applied economists*. Princeton University Press, 1992.

[18] R. B. Myerson, *Game theory: analysis of conficts*. Harvard University Press, 1991.

[19] A. Mas-Colell, M. D. Whinston, and J. R. Green, *Microeconomic theory*. Oxford University Press, 1995.

[20] A. Rubinstein, *Modeling bounded rationality*. The MIT Press, 1998.

[21] R. Dash, N. Jennings, and D. Parkes, "Computational-mechanism design: A call to arms," *IEEE Intelligent Systems*, pp. 40–47, November 2003.

[22] N. Nisan and A. Ronen, "Algorithmic mechanism design," *Games and Economic Behavior*, vol. 35, pp. 166–196, 2001.

[23] I. Gilboa and A. Matsui, "Social stability and equilibrium," *Econometrica*, vol. 59, no. 3, pp. 859–867, May 1991.

[24] M. Kandori and R. Rob, "Evolution of equilibria in the long run: A general theory and applications," *Journal of Economic Theory*, vol. 65, no. 2, pp. 383–414, April 1995.

[25] H. L. Cole and N. Kocherlakota, "Finite memory and imperfect monitoring," *federal Reserve Bank of Minneapolis Working Paper*, 2000.

[26] F. Gul and E. Stacchetti, "The English auction with differentiated commodities," *Journal of Economic Theory*, pp. 66–95, May 2000.

[27] D. C. Parkes and L. H. Ungar, "An ascending-price generalized Vickrey auction," *Harvard University, Tech. Rep.*, 2002.

[28] J. Kalagnanam and D. C. Parkes, *Supply chain analysis in the ebusiness era*. Kluwer, 2002.

[29] D. M. Topkis, *Supermodularity and complementarity*. Princeton University Press, 1998.

[30] D. Monderer and L. Shapley, "Potential games," *Games and Economic Behavior*, vol. 14, pp. 124–143, 1996.

[31] P. Milgrom and J. Roberts, "Rationalizability, learning and equilibrium in games with strategic complementarities," *Econometrica*, vol. 58, no. 6, pp. 1255–1277, Nov. 1990.

[32] ——, "Adaptive and sophisticated learning in repeated normal form games," *Games and Economic Behavior*, 1991.

[33] D. Funderberg and D. M. Kreps, "A theory of learning in games," MIT Press, 1998.

[34] C. U. Saraydar, N. B. Mandayam, and D. J. Goodman, "Efficient power control via pricing in wireless data networks," *IEEE Trans. on Communications*, vol. 50, no. 2, pp. 291–303, Feb. 2002.

[35] E. Altman and Z. Altman, "S-modular games and power control in wireless networks," *IEEE Trans. on Automatic Control*, vol. 48, no. 5, pp. 839–842, May 2003.

[36] J. Huang, R. Berry, and M. L. Honig, "Distributed interference compensation in wireless networks," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 5, pp. 1074–1084, May 2006.

[37] D. C. Parkes, "Iterative combinatorial auctions: Achieving economic and computational efficiency," Ph.D. dissertation, University of Pennsylvania, 2001.

[38] V. Krishna, *Auction theory.* Academic Press, 2002.

[39] R. Wilson, "Auctions of shares," *Quarterly Journal of Economics*, vol. 93, pp. 675–698, 1979.

[40] L. Ausubel and P. Cramton, "Demand reduction and inefficiency in multi-unit auctions," manuscript, University of Maryland, 1998.

[41] C. Maxwell, "Auctioning divisible commodities: A study of price determination," Ph.D. dissertation, Harvard University, 1983.

[42] K. Back and J. F. Zender, "Auctions of divisible goods: On the rationale for the treasury experiment," *Review of Financial Studies*, vol. 6, pp. 733–764, 1993.

[43] J. J. D. Wang and J. F. Zender, "Auctioning divisible goods," *Economic Theory*, no. 19, pp. 673–705, 2002.

[44] A. Hortasu, "Mechanism choice and strategic bidding in divisible good auctions: an empirical analysis of the Turkish treasury auction market," manuscript, Stanford University, 2000.

[45] G. Federico and D. Rahman, "Bidding in an electricity pay-as-bid auction," *Journal of Regulatory Economics*, 2003.

[46] R. Johari and J. N. Tsitsiklis, "Efficiency loss in a network resource allocation game," *Mathematics of Operations Research*, vol. 29, no. 3, pp. 407–435, Aug. 2004.

[47] S. Yang and B. Hajek, "An efficient mechanism for allocation of a divisible good," submitted to *Math Operation Research*, 2005.

[48] S. Sanghavi and B. Hajek, "Optimal allocation of a divisible good to strategic buyers," working paper, UIUC, 2004.

[49] R. Maheswaran and T. Basar, "Nash equilibrium and decentralized negotiation in auctioning divisible resources," *Group Decision and Negotiation*, vol. 12, no. 5, pp. 361–395, 2003.

[50] R. T. Maheswaran and T. Basar, "Coalition formation in proportionally fair divisible auctions," in *Autonomous Agents and Multi-Agent Systems*, 2003.

[51] ——, "Decentralized network resource allocation as a repeated noncooperative market game," in *Proceedings of the 40th IEEE Conference on Decision and Control*, Orlando, FL, December 2001, pp. 4565–4570.

[52] M. Voorneveld, "Best-response potential games," *Economics Letters*, vol. 66, no. 3, pp. 289–295, March 2000.

[53] J. Neel, J. Reed, and R. Gilles, "Convergence of cognitive radio networks," *Prof. of IEEE Wireless Communications and Networking Conference*, 2004.

[54] J. Hicks, A. MacKenzie, J. Neel, and J. Reed, "A game theory perspective on interference avoidance," *Proc. of IEEE Global Telecommunications Conference*, 2004.

[55] G. Scutari, S. Barbarossa, and D. Palomar, "Potential games: a framework for vector power control problems with coupled constraints," *Prof. of IEEE ICASSP 2006*, 2006.

[56] P. Milgrom, *Putting auction theory to work.* Cambridge University Press, 2004.

[57] W. Yu, G. Ginis, and J. M. Cioffi, "Distributed multiuser power control for digital subscriber lines", *IEEE Journal on Selected Areas in Commun.*, vol. 20, no. 5, pp. 1105-1114, June 2002.

[58] H. Yaiche, R. R. Mazumdar, and C. Rosenberg, "A game theoretic framework for bandwidth allocation and pricing in broadband networks", *IEEE/ACM Transactions on Networking*, vol. 8, no. 5, pp. 667-678, Oct. 2000.

[59] D. Grosu, A. T. Chronopoulos, and M.Y. Leung, "Load balancing in distributed systems: an approach using cooperative games", *Proc. of IPDPS 2002*, pp. 52-61, 2002.

[60] G. Owen, *Game theory, 3rd ed.* Academic press, 2001.

[61] W. Rhee and J. M. Cioffi, "Increase in capacity of multiuser OFDM system using dynamic subchannel allocation," *Proc. of IEEE VTC'00*, pp. 1085-1089, 2000.

[62] D. Fudenberg and J. Tirole, *Game theory*, MIT Press, Cambridge, MA, 1991.

[63] Z. Han, Z. Ji, and K. J. R. Liu, "Fair multiuser channel allocation for OFDMA networks using Nash Bargaining and coalitions", *IEEE Transactions on Communications*, vol. 53, no. 8, pp. 1366-1376, August 2005.

[64] C. Peng, H. Zheng, and B. Y. Zhao, "Utilization and fairness in spectrum assignment for opportunistic spectrum access", *ACM Mobile Network Applications*, vol. 11, no. 4, pp. 555-576, August 2006.

[65] J. E. Suris, L. DaSilva, Z. Han, and A. MacKenzie, "Cooperative game theory approach for distributed spectrum sharing", in *Proc. of IEEE International Conference on Communications*, Glasgow, Scotland, June 2007.

[66] K. D. Lee, and V. C. M. Leung, "Fair allocation of subcarrier and power in an OFDMA wireless mesh network", *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 11, pp. 2051-2060, Nov. 2006.

[67] H. Park and M. van der Schaar, "Bargaining strategies for networked multimedia resource management," *IEEE Trans. Signal Process.*, vol. 55, no. 7, pp. 3496-3511, Jul. 2007.

[68] Z. Han, Z. Ji, and K. J. Ray Liu, "Power minimization for multi-cell OFDM networks using distributed non-cooperative game approach", *IEEE Global Telecommunications Conference*, 2004.

[69] Z. Han, Z. Ji, and K. J. Ray Liu, "Non-Cooperative Resource Competition Game by Virtual Referee in Multi-Cell OFDMA Networks," *IEEE Journal on Selected Areas in Communications*, vo. 25, no. 6, pp. 1079-1090, August 2007.

[70] R. J. Aumann, "Subjectivity and correlation in randomized strategy", *Journal of Mathematical Economics*, vol. 1, no. 1, pp. 67-96, 1974.

[71] R. J. Aumann, "Correlated equilibrium as an expression of Bayesian rationality", *Econometrica*, vol. 55, no. 1, pp. 1 - 18, January 1987.

[72] S. Hart and A. Mas-Colell, "A simple adaptive procedure leading to correlated equilibrium", *Econometrica*, vol. 68, no. 5, pp. 1127-1150, September 2000.

[73] Z. Han, C. Pandana, and K. J. R. Liu, "Distributive opportunistic spectrum access for cognitive radio using correlated equilibrium and no-regret learning", in*Prof. of IEEE Wireless Communications and Networking Conference*, Hong Kong, China, March 2007.

[74] E. Altman, N. Bonneau, and M. Debbah, "Correlated equilibrium in access control for wireless communications", in *LECTURE NOTES IN COMPUTER SCIENCE*, vol. 3976, 2006.

[75] V. Krishnamurthy, G. Yin, and M. Maskery, "Stochastic approximation based tracking of correlated equilibria for game-theoretic reconfigurable sensor network deployment", *Proc. of IEEE Conference on Decision and Control*, pp. 2051-2056, San Diego, CA, December 2006.

[76] J. Huang, V.G. Subramanian, R. Agrawal, and R.Berry, "Downlink scheduling and resource allocation for OFDM systems," *IEEE Trans. on Wireless*, to appear.

[77] Y. Wu, B. Wang, K.J.R. Liu, and T.C. Clancy, "Repeated Open Spectrum Sharing Game with Cheat-Proof Strategies," *IEEE Trans. on Wireless Communications*, to appear.

[78] S Adlakha, R Johari, and A Goldsmith, "Competition in Wireless Systems via Bayesian Interference Games, " Arxiv preprint, arXiv:0709.0516, 2007.